Random walk or mean reversion? Empirical evidence from the Crude oil price market.

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CHAPTER 1

Introduction

This introductory chapter briefly discusses crude oil and crude oil price. The chapter also outlines the background of the study, statement of the research problem, objectives and significance of the study. Finally, the chapter also gives the research outline.

1.1 Background to the study

The thrust of the study is to investigate whether crude oil price is mean reverting or follows a random walk process. Crude oil presents an interesting case because; it is the fundamental driver of most economic activities in the world. Crude oil is vital in many industries and of great importance to the maintenance of an industrialized modern economy. Crude oil is an essential commodity for all nations, since it is the driving force of the economies. Higher crude oil prices have a direct impact on macroeconomic variables such as; inflation, Gross Domestic Product (GDP), investments, recessions, and other macro-economic variables (Cheong, 2009). Crude oil prices are related to the global financial markets, including contracts, options, risk management and other related financial derivatives.

It is thus important to investigate whether oil price predictions can be done with accuracy or not. Forecasting crude oil future prices remain one of the biggest challenges facing statisticians and econometricians. Some researchers find crude oil prices follow a random walk, implying that tomorrow's expected oil prices should be the same as today's value. It is imperative to revisit mean reversion and random walk in the context of crude oil as it has serious implication on modeling crude oil prices.

Bernard *et al* (2008) argues that research on crude oil price dynamics for modeling and forecasting has brought forth several unsettled issues. Although, statistical support is claimed for various models of price paths, yet many of the competing models differ importantly with respect to their fundamental properties. One such property is mean reversion. Pindyck (1999) says that unit root tests are inconclusive in the analysis of real prices observed on yearly basis. The author expresses the minimum sample size for which Dickey-Fuller test is significant given a stationary autoregressive data generating process in terms of autocorrelation coefficient. Pindyck (1999) concluded that due to the persistence characteristic crude oil price, a very long and practically unavailable series is required to perform reliable tests. While, Bernard *et al* (2008) concluded that structural discontinuities should be accounted for in examining stochastic models for crude oil prices or for returns i.e. whether one adopts a unit root or a mean reverting model.

It is of importance to briefly discuss this energy commodity whose pricing is central to this research essay.

1.2 Crude oil

Crude oil is a naturally occurring toxic flammable liquid consisting of a complex mixture of hydrocarbons of various molecular weights, and other organic compounds, that are found in geological furmenty beneath the Earth's surface.

The end products of crude oil are:

- various fuels
- lubricants (motor oils and greases).
- wax, used in packaging of frozen foods etc.
- sulfuric acid for making fertilizers and other important solvents.
- petroleum coke, used in carbon products.
- paraffin wax etc

1.3 Crude oil price

A market linked pricing is the main method for pricing crude oil in the international trade. The price markers are Brent, West Texas International (WTI) and Dubai/Oman. There are mainly two pricing systems of crude oil namely spot price and futures. These pricing systems of crude oil differ on the delivery period of crude oil. Spot price is the crude oil price per barrel (159 litres) of WTI / light crude oil or Brent for immediate delivery. Futures price is also crude oil price per barrel of WTI / light crude oil or Brent for delivery at a period greater than one week (Wikipaedia/org).

1.4 Statement of the Research problem

This study investigates whether crude oil price is mean reverting or follows a random walk process.

1.5 Aims and Objectives of the Study

The main aim and objective of the study is to use the two approaches namely, The Augmented Dickey-Fuller tests and Garch model with time-varying properties approach, to investigate mean reversion and random walk processes in crude oil price.

This can be achieved by :

- plotting and analyzing the time series of crude oil price from 1980 to 2010.
- using the Autocorrelation Function (ACF) and the Augmented Dickey-Fuller Test, to test for a unit root for data segmented into periods namely 1980 to 1995 and 1995 to 2010. The reasons for segmenting the data will be revealed later in the research.
- using the model with time –varying properties to investigate mean reversion and random walk.
- giving recommendations for further study.

1.6 Significance of the study

The results and recommendations of this research will be of interest to statisticians, researchers, econometricians, industry and government decision makers and other interested stakeholders like energy investors, in terms of modelling and predicting crude oil price.

1.7 Research Layout

The study is organized as follows: chapter 2 gives an overview of the random walk models and related literature. Chapter 3 describes the empirical methods used in this study. In chapter 4, the data and results are reported and discussed. The last chapter provides a conclusion.

CHAPTER 2

Literature Review

This chapter discusses the random walk process and mean reversion. It also gives some documented researches on investigations on whether crude oil price is mean reverting or a random walk process.

Considering an Auto-Regressive model of order 1, AR (1) model, for crude oil prices :

$P_t = \alpha + \varphi P_{t-1} + \mu_t$, where P_t in the crude oil price, α and φ are constants, μ_t is a white noise error term. i.e. $\mu_t \sim N(0, \sigma^2)$.

A random walk is a special case of an AR (1) model with $\varphi = 1$. A random walk is a classic example of a non stationary stochastic process. Asset prices such as stock prices or exchange rates follow a random walk, thus are non stationary (Gujarati, 2004). There are mainly two types of random walk models namely; random walk without a drift (i.e. no constant or intercept term) and random walk with a drift (i.e. a constant term is present).

2.1. Random Walk Model with no drift parameter

A time series (P_t) is a random walk with no drift if it satisfies the following equation

$$P_t = P_{t-1} + \mu_t \qquad (2.1)$$

The random walk implies that the value of P at time t is equal to its value at time $(t-1)$ plus a random shock (μ_t). The sequence $\{\mu_t\}$ is white noise error terms with mean zero and variance σ^2. P_t is the price of crude oil at time t.

A random walk model with no drift is an AR (1) model with $\alpha = 0$ and $\varphi = 1$.

It is easy to see that $P_t = P_0 + \sum \mu_t$ and $E(P_t) = E(P_0 + \sum \mu_t) = P_0, Var(P_t) = t\sigma^2$

The mean of $P_t$is constant for a random walk without drift but the variance increases with time t. The increasing variance violates a condition of weak stationarity. Thus, the random walk model without drift is a non stationary stochastic process. A random walk model remembers the shocks (random errors) forever and is said to have an infinite memory (Gujarati, 2004).

Equation (2.1) can be written as:

$$(P_t - P_{t-1}) = \Delta P_t = \mu_t \tag{2.2}$$

Thus while P_t is non stationary; its first difference is stationary since μ_t is a stationary white noise process i.e. $\mu_t \sim N(0, \sigma^2)$

In other words, the first difference of a random walk time series is stationary.

2.2 Random Walk Model with drift (constant term)

Random walk with a drift is also a special form of an AR (1) model:

$$P_t = \alpha + \varphi P_{t-1} + \mu_t \text{ , where } \alpha \neq 0 \text{ and } \varphi = 1$$

Thus can be written as:

$$P_t = \alpha + P_{t-1} + \mu_t \tag{2.3}$$

where α is the drift parameter.

Equation (2.3) can be written as:

$$P_t - P_{t-1} = \Delta P_t = \alpha + \mu_t \tag{2.4}$$

It shows that P_t drifts upwards or downwards, depending on α being positive or negative.

The expected value of crude oil price at time t i.e.

$$E(P_t) = t\alpha + P_0 \tag{2.5}$$

and variance of crude oil price at time t i.e.

$$Var(P_t) = t\sigma^2 \tag{2.6}$$

Thus the mean and variance of the random walk with a drift increases with time t, again violating the conditions of weak stationarity. In short a random walk model with or without drift parameter is a non stationary process.

2.3 Mean Reversion

If a time series is weakly stationary, its mean and variance are constant and the auto covariance depends on the time lag. Such a time series will tend to return to its mean (Mean Reversion) and fluctuations around this mean (measured by its variance) will have a broadly constant amplitude. Mean reversion of prices imply that when prices goes up there is a normal level to which they will eventually return. A mean reverting market is one where rises are more likely to follow a market fall, and a fall is more likely to follow a rise. Hence, if prices have recently been above the long run average, then it is expected that prices to be lower than average over the next few periods, so that average prices revert back towards their long run trend level.

There appears to be some evidence of mean reversion, but the evidence rests heavily on the aftermath of a small number of dramatic crashes. After a major crash, markets are expected to revert to their former level after sufficient time (Taylor and Doren, 2008).

It is expected that some mean reverting force will pull prices back to some normal range, over the long run to form some stationary distribution. Prices are not independent from one year to the next. Times of high Prices tend to bunch together, i.e. the models are autoregressive. In particular then, it is not expected that a white noise process would be a good model for prices. Instead some dependence on previous values must be built in the model, and this is what autoregressive models do. For instance this version of an AR (1) process $P_t - \mu = \varphi(P_{t-1} - \mu) + \mu_t$, is stationary if $|\varphi| < 1$, where μ is the mean of the process. Ignoring the white noise error term, the distance of P_t from its long run mean μ is φ times the previous distance. If $|\varphi| < 1$ then the distance is decreasing so that the process is being pulled back to the mean since:

$$P_t = \mu + \varphi(P_{t-1} - \mu) + \mu_t \ , \qquad (2.7)$$

If $|\varphi| \geq 1$, the process is not mean reverting. If $\varphi = 1$, the process follows a random walk.

2.4 Unpredictability of a Random walk

For a random walk process, the $l-$step ahead forecast of equations (2.1) and (2.3) at the forecast horizon h is

$$P_h(l) = E[P_{h+1}|P_h, P_{h-1} \dots \dots .] = P_h \tag{2.8}$$

which is the price of the stock at the forecast origin.

For all forecast horizons; point forecasts of a random walk are simply the value of the series at the forecast origin. Therefore the process is not mean reverting.

Consider the random walk with drift parameter $\alpha = 0$ and initial value $P_0 = 0$:

$P_t = \alpha + P_{t-1} + \mu_t$

The moving average representation of the random walk model in equation (2.1) is

$$P_t = \mu_t + \mu_{t-1} + \mu_{t-2} + \cdots \mu_1$$

This representation has the following practical implication.

The, $l-$step ahead forecast error is

$$e_h(l) = \mu_{h+l} + \cdots + \mu_{h+1}$$

So that Var $[e_h(l)] = l\sigma^2$, which diverges to infinity as $l \to \infty$. The length of an interval forecast of P_{h+l} will approach infinity as the forecast horizon increases. This result says that the usefulness of point forecast $\hat{P}_h(l)$ diminishes as l increases, which implies that the random walk model predictions are not useful (Tsay, 2002).

2.5 Implications of the random walk and mean reversion

The fact that a simple random-walk model could be used to model crude oil price, implies that the best prediction price at time $t + 1$ should be the price at time t. Essentially the random walk model implies successive crude oil price movements should be independent. The random walk is closely associated with the efficient market hypothesis, as the more efficient the market, the more random the price changes. An efficient market means that stock market returns (or prices) cannot be predicted by observing historical observations and this is what would characterize a crude oil market which follows a random walk.

However, mean reversion (or deviations from the random walk hypothesis) to stock prices has been known for some time and some of the mean reversions are mentioned below:

Negative serial correlation indicates higher than average returns would be followed by lower than average returns. If the random walk model were true, we would expect zero serial correlation. Lo and MacKinley (1999) concluded that stock prices short-run serial correlations are not zero. In the long run there is evidence of negative autocorrelation giving rise to mean reversion.

There are **seasonal trends** in the stock market, especially at the beginning of the year and the end of the week.

Prices sometimes **over/under react** to economic or global events such as earnings announcements and Sovereign credit ratings. Subsequent market corrections are known to occur (Dupernex, 2007).

2.6 Related Literature

Pilipovic (1997) cited in Sharma (2008) argued that evidence suggests that log price is mean reverting or stationary. The process generating such a series would not have a unit root, thus the coefficient for P_{t-1} in the log price random walk equation would be less than one. Tayor and Doren (2008), concluded that crude oil prices move akin to a random walk without a drift term ($\alpha = 0$). The best predictor of the future oil price is the present oil price.

Geman (2007) used the following model to investigate the statistical properties of crude oil prices:

$$p_t = \varphi p_{t-1} + \mu_t$$

to check whether the coefficient φ is significantly different from 1, where p_t is the log of crude oil price. The null hypothesis is the existence of a unit root (i.e. $\varphi = 1$). A $p\ value$ smaller than 0.05 rejected the null hypothesis with a confidence level higher than 0.95. Thus the bigger the $p\ value$, the more the random walk model is validated. Geman (2007) used the West Texas International (WTI) oil spot prices over the period January 1994 to 2004. The author got an ADF

$p\ value of$ 0.651 for spot prices of oil prices for the period January 1994 to October 2004. This result rejects the mean reversion assumption over the whole period and confirms that log crude oil price follows a random walk during that period.

However, Geman (2007) noted that a mean reversion pattern of crude oil prices prevails for a shorter period from 1994 to 2000, and it changes into random walk as of 2000. Actually, Geman (2007) concluded that there is need to mix mean reversion for spot crude oil prices towards a long term value of oil prices driven by a Brownian motion. According to the author, the following three state variable model incorporates stochastic volatility:

$$dS_t = a(L_t - S_t)S_t dt + \sigma_t S_t dW_t^1,$$

$$dy_t = \alpha(b - y_t)dt + \eta\sqrt{y_t}dW_t^2, \quad \text{where } y_t = \sigma_t^2$$

$$dL_t = \mu L_t dt + \xi L_t dW_t^3,$$

Where S_t is the spot price, L_t is long term value, W_t is the standard Brownian motion on a probability space(Ω, F, P), the Brownian motions are positively correlated. The positive correlation between W^1 and W^2 accounts for the "inverse leverage" effect that prevails for commodity prices (in contrast to the "leverage effect" observed in the equity markets), whereas the positive correlation between W^1 and W^3 translates the fact that news of depleted reserves will generate a rise in both daily and long term oil prices (Geman, 2007).

Bessembinder *et al* (1995) analyses the relation between oil price levels and slope of the futures term structure defined by the difference between a long maturity future contract and its first nearby. Assuming that future prices are unbiased expectations (under the real probability measure) of future spot oil prices, an inverse relation between prices and the slope constitutes evidence that investors expect mean reversion in spot prices, as it implies a lower expected future spot prices when prices rise. The authors concluded the existence of mean reversion of oil price over the period 1982-1999, however the same computations conducted over the period of 2000-2005 leads to inconclusive results (Geman, 2007). Thus more work remains to be done in this period and beyond.

Crude oil prices are characterized by the phenomenon known as volatility clustering, i.e. periods in which they exhibit wide swings for an extended time period followed by a period of comparative tranquility. According to Engle and Patton (2000), volatility is mean reverting. Mean reversion in volatility is interpreted as meaning that there is a normal level of volatility to which volatility will eventually return. More precisely, mean reversion in volatility implies that current information has no effect on the long run forecast. Thus it is of great importance to find out if oil price is mean reverting or a random walk process.

CHAPTER 3

Methodology

This chapter outlines the methodology applied in analyzing and discussing the data. There are many tests which can be used to check if data follows a random walk process. In this study, the Auto Correlation Function (ACF) and the Augmented Dickey – Fuller unit root test (ADF) , will be used to test for stationarity and make inference on mean reversion and the random walk. A Garch approach test with time varying parameters will also be used to do the same test.

3.1 Auto Correlation Function (ACF)

The Auto Correlation Function **(**ACF) at lag k for crude oil returns, denoted by ρ_k is defined as:

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{covariance\ at\ lag\ k}{variance} \quad ,-1 \leq \rho_k \leq 1 \tag{3.1}$$

A plot of $\rho_k$against k is known as a correlogram. For a purely white noise process the auto correlations at various lags hovers around zero. This property shows the prices are a stationary time series. A slow decaying ACF is an indicator of large characteristic root and a unit root time series. A random walk series has very high autocorrelation coefficients at various lags. In between the two extremes of the white noise process and the random walk the ACF drops fast after a given number of lags to give the order of the autoregressive process (Enders, 2004).

3.2 The Augmented Dickey-Fuller Unit Root Test (ADF)

The ADF consist of estimating the regression coefficient of P_t on P_{t-1}. If this coefficient is significantly below 1, it means that the process is mean revering: if it is close to 1, the process is a random walk (Geman, 2007: p 233)

The test is based on the standard Dickey-Fuller Test for on an AR (1) process:

$$P_t = \varphi P_{t-1} + \mu_t \tag{3.2}$$

Subtracting P_{t-1} on both sides gives:

$$P_t - P_{t-1} = \varphi P_{t-1} - P_{t-1} + \mu_t$$

giving,

$$P_t - P_{t-1} = (\varphi - 1)P_{t-1} + \mu_t$$

thus can be written as: $\Delta P_t = \beta P_{t-1} + \mu_t$ (3.3)

where $\beta = \varphi - 1$, P_t is the oil price at time t, $\Delta P_t = P_t - P_{t-1}$ and $\mu_t \sim N(0, \sigma^2)$.

There are variations of equation (3.3). Firstly, equation (3.3) tests for the presence of a pure random walk with no trend or intercept.

The second form is as follows:

$$\Delta P_t = a_o + \beta P_{t-1} + \mu_t \quad (3.4)$$

It test for a random walk with a drift term, where a_0 is the drift term.

Lastly, equation (3.5):

$$\Delta P_t = a_o + \beta P_{t-1} + a_1 t + \mu_t \quad (3.5)$$

tests for a random walk with both a drift, a_0, and a linear trend term, a_1. The difference between the three regressions concerns the presence of deterministic elements a_o and $a_1 t$. The parameter of interest in all the regression equations is β, if $\beta = 0$, the $\{P_t\}$ sequence contains a unit root (Enders, 2004).

The following formal hypothesis is tested:

Hypotheses

H_0: $\beta = 0$ (series is non-stationary)

H_1: $\beta < 0$ (series is stationary)

Test Statistic:

$$t_\beta = \frac{\hat{\beta}}{se_{(\hat{\beta})}} \quad (3.6)$$

where $\hat{\beta}$ is the estimate of β and $se_{(\hat{\beta})}$is the coefficient standard error.

The ADF test constructs a parametric test for higher order correlation by assuming that the series follows an AR (p) process and adding p lagged difference terms of the dependent variable P to the right side of the test regression:

$$\Delta P_t = a_o + cX_t + \beta P_{t-1} + \beta_1 \Delta P_{t-1} + \beta_2 \Delta P_{t-2} + \cdots \ldots \ldots . + \beta_p \Delta P_{t-p} \quad (3.7)$$

where cX_t is an exogenous variable.

In this study, test regression of lag 1 is considered.

$$\Delta P_t = a_o + \beta P_{t-1} + a_1 t + \mu_t$$

This augmented specification is then used to test the hypothesis using the test statistic (3.6). From equation (2.2 and 2.4), the first difference of a random walk model is stationary. The ACF and ADF is used to test for stationarity of the original series. If the plot of first difference of the series is quite random (no predictable pattern) then it is stationary and the original series is non stationary i.e. random walk (Gujarati, 2004).

Drawbacks:

This test however has its own limitations such as the researcher must choose whether to include exogenous variable in the test regression or include more than one lag. The test has low power in local stationary alternatives in small samples, especially when the time series under investigation are near-integrated process. The ADF test tends to accept the null unit root more frequently than is warranted. That is the test may find a unit root even when none exist (Enders, 2004).

3.3 A Simple model for log returns

We define the natural logarithmic return (simply log return) of crude oil at time t as:

$$r_t = Log(P_t/P_{t-1}) = Log(P_t) - Log(P_{t-1}) = p_t - p_{t-1}$$

where P_t is the price of crude oil at time t.

The simplest model which can be used to test for the random walk is the simple auto-regressive (AR (1)) model namely:

$$r_t = \beta_0 + \beta_1 r_{t-1} + \mu_t \tag{3.8}$$

where $r_t = p_t - p_{t-1}$, is the log return of crude oil price, β_0 and β_1 are the parameters that need to be estimated and $\mu_t \sim IID\ (0, \sigma^2)$, $p_t = \log{(P_t)}$ is the natural logarithm of the price of crude oil at time t.If the crude oil price follows a random walk, $\beta_1 = 0$ and so

$$p_t = \beta_0 + p_{t-1} + \varepsilon_t, \tag{3.9}$$

the random walk with drift parameter β_0.

The natural logarithmic transformation reduces the impact of heteroskedasticity that may be present when you have large data sets with high frequency. The transformation also ensures that predicted crude oil price is positive when anti-logs are taken. The model however does not cater for changing volatility.

Three versions of the random walk model are distinguished by Cambell, Lo and MacKinlay (1997) and also cited in Jefferis and Smith (2005:p.59) which depend on the assumptions of the error term, namely ε_t. Under the first model, the error terms are independently and identically distributed with a zero mean and constant variance, denoted by $\varepsilon_t \sim IID\ (0, \sigma^2)$. In the second model, the error terms are independent but not identically distributed, which allows for unconditional heteroscedasticity in the ε_t or $\varepsilon_t \sim NID\ (0, \sigma_t^2\ )$. The problem of heterogeneously distributed processes is relevant, since crude oil prices have been found to display heteroscedasticity. In the third random walk model, the error terms are uncorrelated and neither independent nor identically distributed as mentioned in the research of Jefferis and Smith (2005). This paper will also focus on the third model, with volatilities changing over time.

Equation (3.8) has constant parameters and the error terms are assumed to follow the usual classical assumptions. With financial markets, the assumption of constant variance may be inappropriate as empirical evidence frequently finds that returns have a variance which changes systematically. Equation (3.8) cannot readily capture gradual deviations towards/ from the random walk over successive observations.

3.4 Garch approach with time varying parameters

Emerson et al (1997) and Zalewska-Mitura and Hall (1999) have developed, using a Garch approach, a test with time-varying parameters which detects changes towards/from the random walk where the error process does not have a full set of NIID properties. The model checks for changes towards/from the random walk and allows the error process to deviate from the property of being normally independent and identically distributed. The test does 3 things: first, it checks for the random walk; second, it detects changes from/towards the random walk, and third, it will operate with stochastic series for which the error process might not have a full set of NIID properties.

The test is based on the following set of equations to constitute the model:

$$r_t = \beta_{0t} + \beta_{1t} r_{t-1} + \delta\sigma_t^2 + \mu_t \tag{3.10}$$

$$\mu_t | \psi_{t-1} \sim N(0, \sigma_t^2) \tag{3.11}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 \tag{3.12}$$

$$\beta_{0t} = \beta_{0t-1} + v_{0t}\ ; \qquad v_{0t} \sim N(0, \sigma_{\beta_0}^2) \tag{3.13}$$

$$\beta_{1t} = \beta_{1t-1} + v_{1t}\ ; \qquad v_{1t} \sim N(0, \sigma_{\beta_1}^2) \tag{3.14}$$

in which σ_t^2 is the conditional variance of the error term ε_t , a Garch(1,1) model. ψ_t is the information set available at time t. α_0, α_1 and γ_1 are parameters needed to model the changing volatility. This model has three important characteristics. First, the intercept, β_{0t} and slope coefficient, β_{1t}, can change through time. However, the special cases where either or both of these are constant are also included. Secondly, this model incorporates an error process in which the variance changes systematically over time. Thirdly, the mean of the log return depends on its conditional variance (level of risk).The basic insight is that risk-averse investors will require compensation for holding a risky asset such as crude oil. A maximum likelihood search procedure with a standard Kalman filter is used to estimate the model with equation (3.10), the measurement equation, and the set of equations given by (3.12), (3.13) and (3.14), the state equations. The Kalman filter sequentially updates coefficient estimates and generates the set of β_{it}'s, $i = 0,1$ and $t = 1 \dots T$ and their standard errors. If the crude oil log returns follows a random walk with no drift, then a $100(1-\alpha)\%$ confidence band for each of β_{0t} and β_{1t} should contain zero. The focus of this study is to find out if crude oil prices follow a random walk or is mean reverting.

3.5 Extending the model

Zalewska-Mitura and Hall (1999) have an extension to the model in section 3.4.

The test is based on the following set of equations:

$$r_t = \beta_{0t} + \sum_{i=1}^{p} \beta_{it} r_{t-i} + \delta\sigma_t^2 + \mu_t \tag{3.15}$$

$$\mu_t | \psi_{t-1} \sim N(0, \sigma_t^2) \tag{3.16}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \gamma_1 \sigma_{t-1}^2 \tag{3.17}$$

$$\beta_{it} = \beta_{it-1} + v_{it} \, ; \qquad v_{it} \sim N(0, \sigma^2_{\beta_i}) \tag{3.18}$$

Such a model can again be modeled using the standard Kalman Filter. The parameters required to estimate time-paths of β_{it}, and δ, α_0, α_1, γ_1 and all p values of $\sigma^2_{\beta_i}$ can be found by maximizing the Likelihood Function. If the series r_t is a random walk, the $100(1-\propto)\%$ confidence bands for each of the β_{it}'s must contain zero.

3.6 Building AR (p)models

An important step in the model identification process is to find the order of the auto-regressive process for the log returns. There are three basic steps to follow to fit AR (p) models to time series data. These steps involve plotting the data, possibly transforming the data, identifying the dependence orders of the model, parameter estimation, and diagnosis and model choice. The ACF is used to identify the order of the model.

3.6.1 The Box-Jenkins strategy

This technique is for modelling stationary time series. The three-stage method will be used for modelling the AR (p) models. It involves the following stages:

Stage 1: Identification

A visual examination of the time series plot, the Autocorrelation function (ACF) is done. Time series plot provides useful information concerning outliers, missing values, structural breaks and trends in the data. The correlograms associated with ACF is often good visual diagnostics tools. Non-stationary time series will be transformed and differenced to achieve stationarity.

Stage 2: Estimation

The tentative models are estimated and examined in line with the principle of parsimony since a parsimonious model fits the data well without incorporating any needless coefficients (Nochai and Nochai, 2006). As a result, Akaike information criterion and Schwartz come in handy as they are approximate measures of the overall fit of the model.

Stage 3: Diagnostic checking

The residuals from the tentative model are examined to find if they approximate a white noise process. Outliers and evidence of periods in which the model does not fit the data well are checked by looking at the plot of residuals. The ACF and PACF of the residuals of the estimated model are used to check for autocorrelation of the residuals to determine whether any or all of the residuals autocorrelations or partial autocorrelations are statistically significant and hence the Ljung-Box Q statistic will be used. Thus, the adequacy of the models is tested using Ljung-Box statistics for the residuals and squared standardized residuals. The Ljung-Box Q statistic at lag $k = 1,2 \ldots . s$, test for the null hypothesis that there is no autocorrelation up to order k is:

$$Q = T(T+2)\sum_{k=1}^{s}\frac{\tau^2}{T-k} \qquad (3.19)$$

where: T is the total number of observations and τ^2 is the k^{th} sample autocorrelation, s is the degrees of freedom. Under the null hypothesis, the Q statistic is asymptotically distributed as a χ^2 distribution with number of degrees of freedom equivalent to the number of calculated autocorrelations. The Box-pierce Q - statistic will not be used as it does not work properly or well in moderate large samples such as the one in this study (Gujarati, 2004).

CHAPTER 4

Data and Results

This chapter analyses and discusses data sources, time series plots and examines the ACF correlograms of two data sets and their segments. The data is on crude oil prices. The chapter also discusses the results of the random walk and mean reversion tests (Augmented Dickey-Fuller tests). Lastly results from the Garch model with time-varying parameters approach are also discussed.

4.1 Data Source

There are two data sets available covering different periods. A single continuous data set ranging from 1980 to 2010 was not easy to find. The first data set available is monthly crude oil price data ranging from January 1980 to January 2007 and is quoted in US Dollars. The data is a simple average of three spot crude oil prices namely Brent, West Texas International (WTI) and the Dubai Fatch. The second data set is also monthly crude oil price and in US dollars and ranges from January 1988 to November 2010. The data is the monthly average crude oil price for three crude oil purchasers in the Illinois Basin namely County mark Coop, Plains and Bi-Petro. This data set of crude oil prices is from the website http://www.inflationData.com.

The two data sets were used to form two segments of data namely, 1980 to 1995 and 1995 to 2010 segments, for reasons to be clearly highlighted later in the chapter.
Both data series are also transformed into monthly log returns series by taking the first difference in logarithms of the prices to give the log returns.

4.2 Time series plot of the data.

The time series plots of the two data sets are shown below:

Crude Oil Prices for the period 1980 to 2007

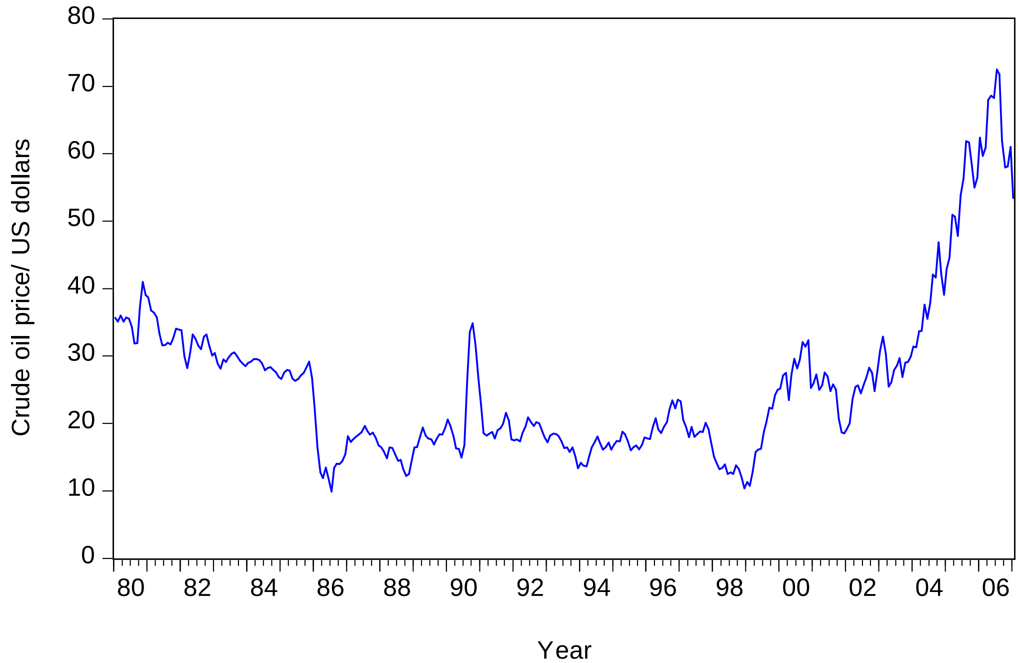

Figure 4.1: Time series plot of Crude oil prices for the period 1980 to 2007

Figure 4.1 shows a time series plot of the first data set. There seems to be a general decrease of crude oil prices from 1980 to 1985. This period is followed by a generally low crude oil prices up to 2001, then, followed by a period from 2002 to 2007 of increasing crude oil prices.

Crude oil prices for the period 1988 to 2010

Figure 4.2 shows a time series plot of monthly crude oil prices for the period 1988 to 2010.

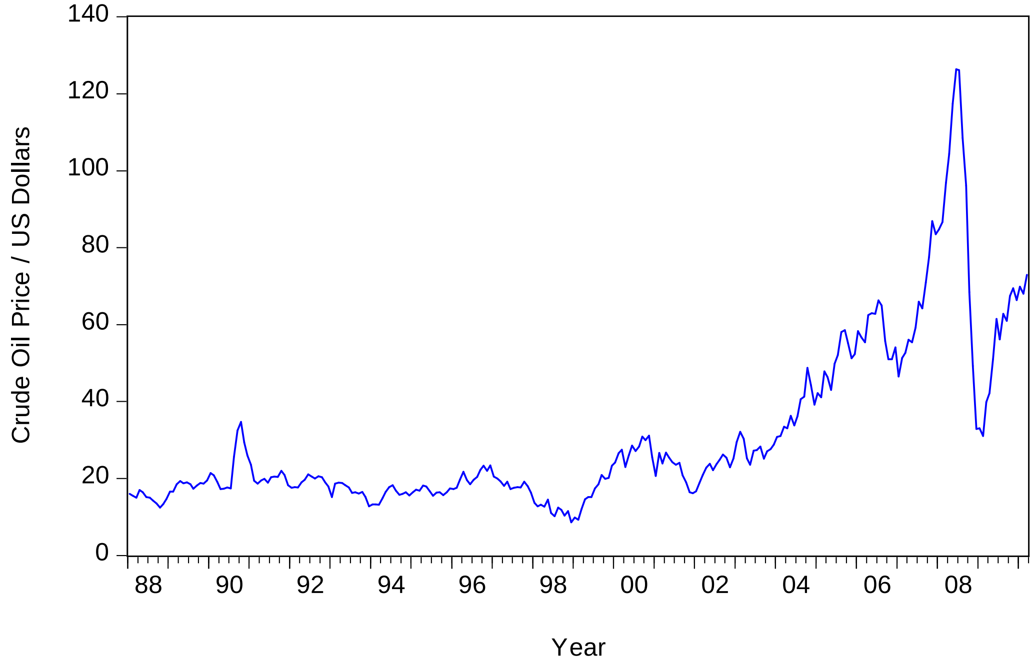

Figure 4.2 Time series plot of Crude oil prices for the period 1988 to 2010

The price of crude oil increased in 1990 mainly due to the invasion of Kuwait and the Gulf war. Figure 4.2 shows an interesting development where the trend seems to dominate the path of the series, especially after 1998. There seems to be a general increase of crude oil prices over these years. However, there are sharp falls in price in 1999 and 2009. In 1999, the fall was attributed to the collapse of Asian Tigers which led to the Asian financial crisis, whereas the fall in oil prices in 2009 is attributed to the USA economic recession. The oil price levels became more volatile and developed a strong upward drift from 2002 to 2007 followed by a sharp fall in 2009 (Sharma, 1998).

4.3 The ACF for crude oil prices for the Period 1980 to 2007

The ACF correlogram of crude oil price for the period from 1980 to 2007 in Figure 4.3 shows an expected feature. The most evident feature of the correlogram is that the autocorrelation coefficients at various lags are very high. The autocorrelation coefficient starts at a very high level at lag 1 and declines very slowly. Thus it seems that crude oil price time series is non stationary.

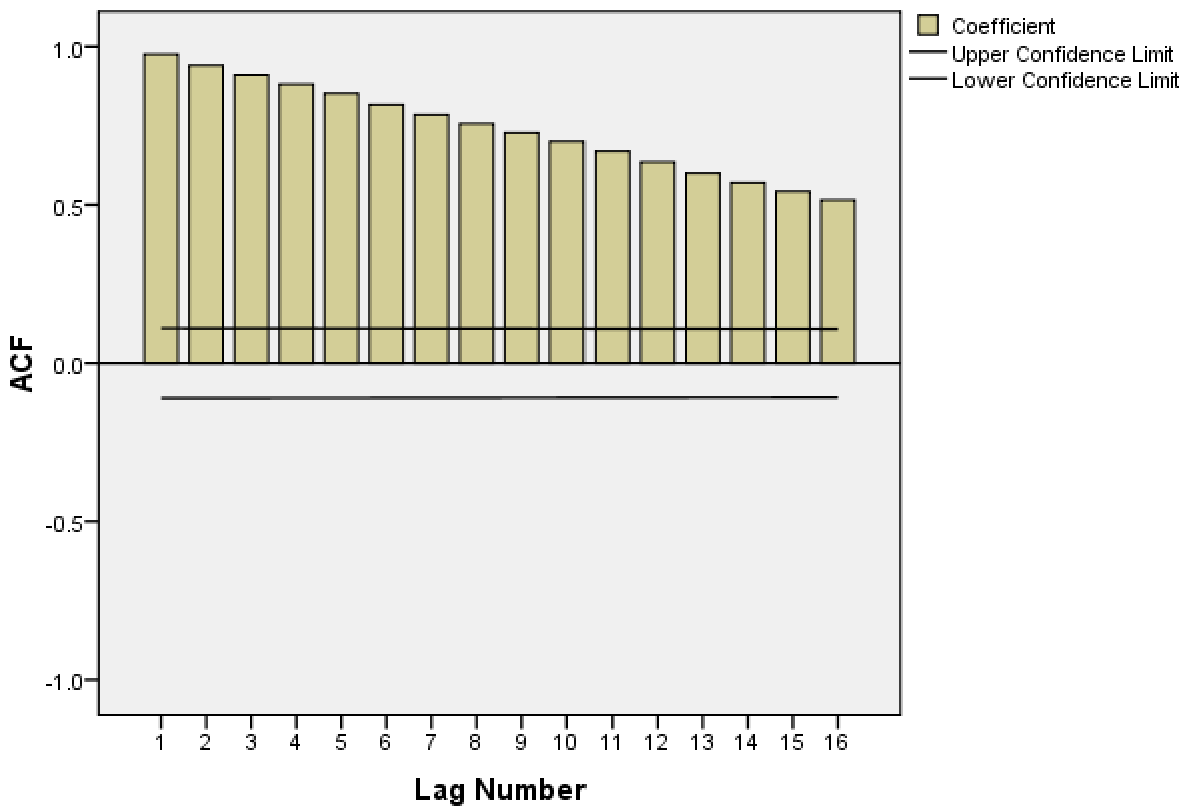

Figure 4.3 Correlogram of crude oil price, 1980 to 2007

4.3.1 The ACF for log crude oil price for the period 1980 to 2007

Consider Figure 4.4 which shows the ACF correlogram of log crude oil price for the period 1980 to 2007. This is a typical correlogram of a non stationary time series. The autocorrelation coefficients are decreasing very slowly suggesting that the log crude oil price is non stationary.

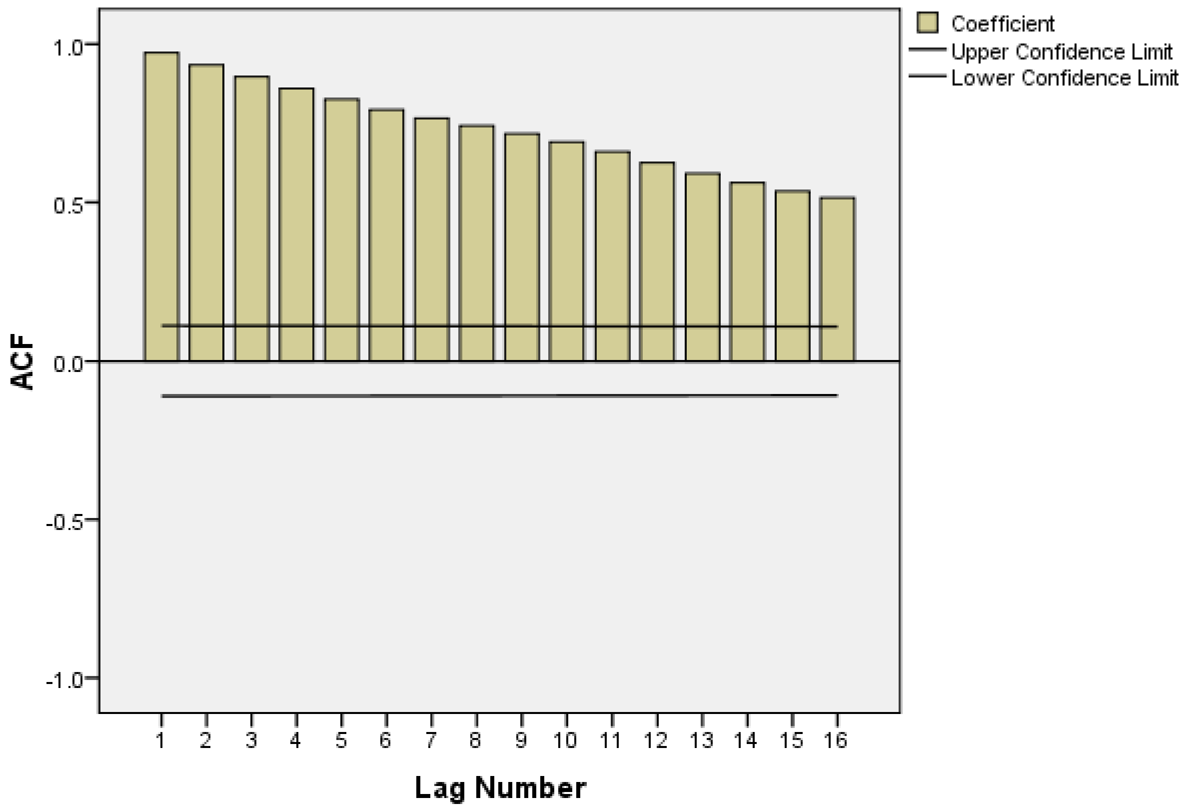

Figure 4.4 Correlogram of log crude oil price, 1980 to 2007

4.4 The Augmented Dickey-Fuller Test for crude oil prices for the Period 1980 to 2007

The ADF test is used to test for random walk or mean reversion for the data set from 1980 to 2007 and conclusions made in line with Geman's (2007) paper.

The model is: $\Delta P_t = \hat{a}_0 + \hat{\beta} P_{t-1} + \hat{a}_1 t$

$H_o: a_0 = 0, a_1 = 0, \beta = 0, series\ is\ non\ stationary\ (has\ a\ unit\ root)$

Table: 4.1 ADF test for crude oil price for the period 1980 to 2007

Test	***Crude oil price***	
	Test statistic	*p value*
ADF	-1.375580	0.8664

Table 4.1 show the results of ADF test on crude oil price data from 1980 to 2007. Since the test statistic of crude oil price is smaller than the critical values at 5% level, the null hypothesis of non-stationary (unit root) is not rejected implying that oil prices are non stationary. This result is consistent with the ACF results. This result implies that crude oil price follows a random walk according to Geman (2007: p 233).

For the log crude oil price, the model is:

$$r_t = \Delta p_t = \hat{a}_0 + \hat{\beta} p_{t-1} + \hat{a}_1 t$$

$H_o: a_0 = 0, a_1 = 0, \beta = 0, series\ is\ non\ stationary\ (has\ a\ unit\ root)$

Table: 4.2 ADF test for log crude oil price for the 1980 to 2007

Test	***Log crude oil price***	
	Test statistic	*p value*
ADF	-1.6717222	0.7617

Consider Table 4.2 showing the result of the ADF test on log crude oil price data from 1980 to 2007. The test statistic of log crude oil price is smaller than the critical value at 5% significance level, the null hypothesis of unit root is not rejected. This result implies that the log crude oil price is non stationary and thus a random walk. This is consistent with the ACF result.

4.5 The ACF for crude oil prices for the period 1988 to 2010

The ACF correlogram of crude oil price for the second data set for the period 1988 to 2010 is shown in Figure 4.5.

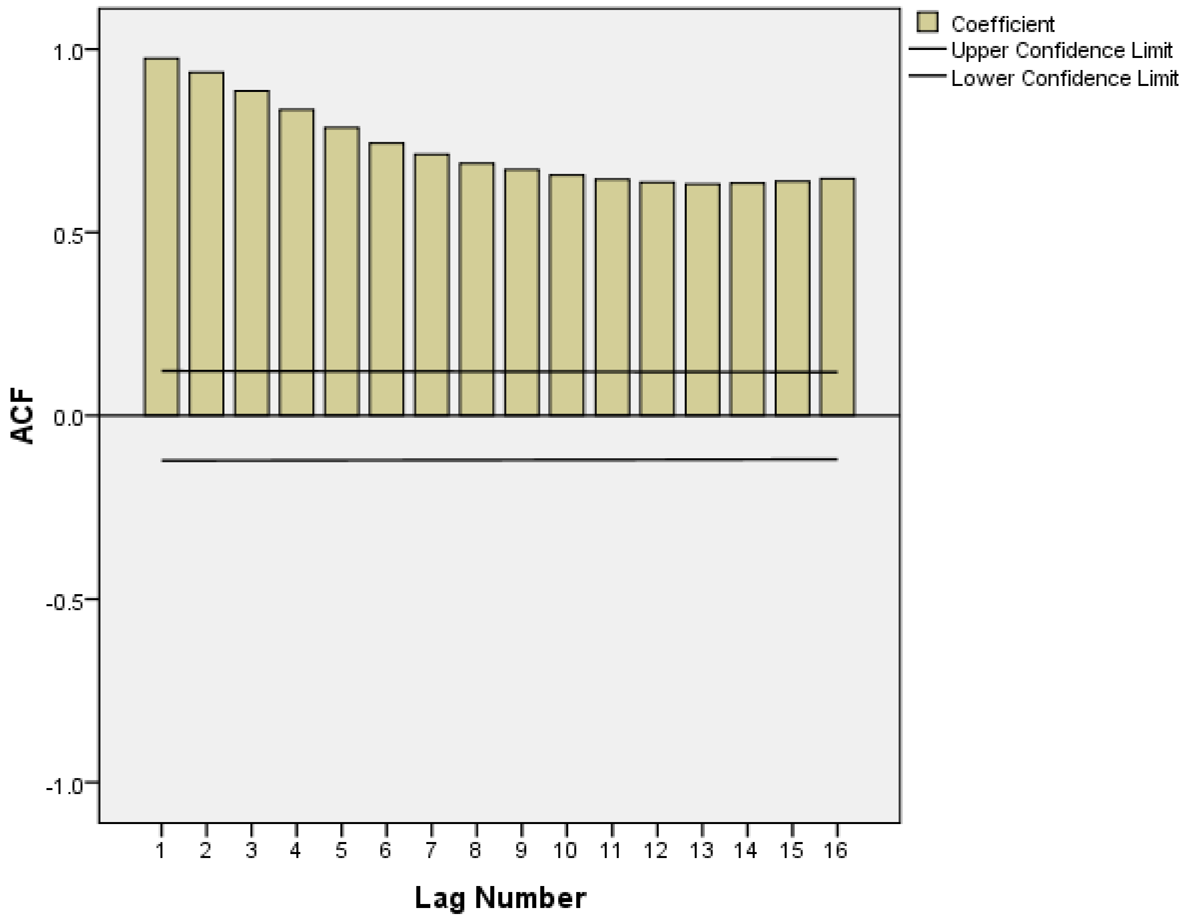

Figure 4.5 Correlogram of crude oil price, 1988 to 2010

Figure 4.5, shows an expected feature. The autocorrelation coefficient starts with very high values and declines very slowly towards zero as the lag length increase. This is a typical correlogram of a non-stationary time series. Thus the crude oil price for the period 1988 to 2010 is also non stationary.

4.5.1 The ACF for log crude oil price for the period 1988 to 2010

The ACF correlogram for the log crude oil price for the period 1988 to 2010 in Figure 4.6 shows that the autocorrelation coefficients for the data are declining as the lag length increases. This is a typical picture of a non stationary time series. Thus the log crude oil price for this period is non stationary.

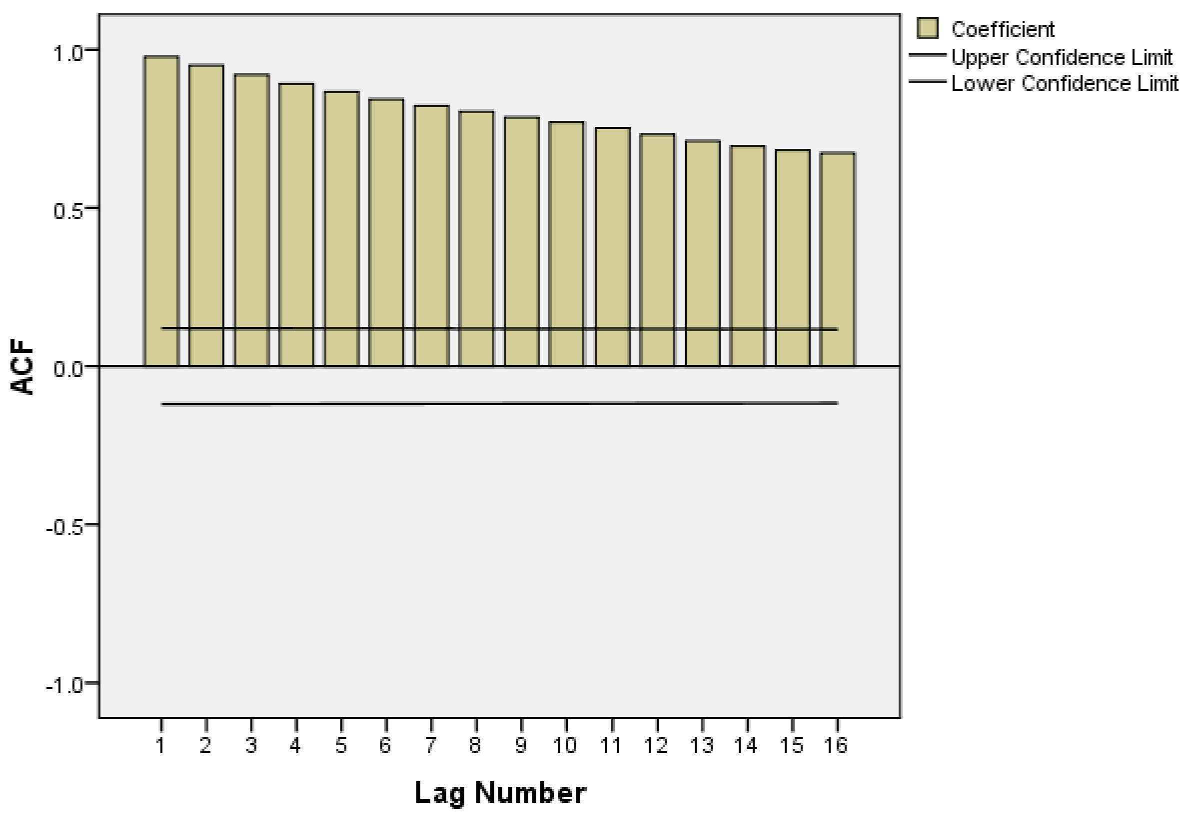

Figure 4.6 Correlogram of log crude oil price, 1998 to 2010

4.5 The Augmented Dickey-Fuller Test for crude oil price for the period 1988 to 2010

The ADF test was used to test for the random walk or mean reversion for the data set for the period 1988 to 2010. The model is:

$$\Delta P_t = \hat{a}_0 + \hat{\beta} P_{t-1} + \hat{a}_1 t$$

$H_o: a_0 = 0\ , a_1 = 0, \beta = 0\ , series\ is\ non\ stationary\ (has\ a\ unit\ root)$

Table 4.3 shows the ADF test of crude oil price for the period 1988 to 2010. The test statistic has a $p\ value = 0.4606$, which is more than 0.05. The null hypothesis of a unit root is not rejected and thus concludes that crude oil price follows a random walk for the period 1988 to 2010 according to Geman (2007).

Table: 4.3 ADF test for crude oil price for the period 1988 to 2010

$\boldsymbol{Test}$	$\boldsymbol{Crude\ oil\ price}$	
	$Test\ statistic$	$p\ value$
$\boldsymbol{ADF}$	-2.248060	0.4606

For the log crude oil price the model is: $r_t = \Delta p_t = \hat{a}_0 + \hat{\beta} p_{t-1} + \hat{a}_1 t$

$H_o: a_0 = 0\ , a_1 = 0, \beta = 0\ , series\ is\ non\ stationary\ (has\ a\ unit\ root)$

Table 4.4 shows the result of the ADF test for crude oil log returns. The test statistic of -2.391138 ($p\ value = 0.3833$) is smaller than the critical value at 5% significance level. This implies that the series of log crude oil price is non stationary (thus a random walk) according to Geman (2007).

Table: 4.4 ADF test for log crude oil price for the period 1988 to 2010

$\boldsymbol{Test}$	$\boldsymbol{Log\ crude\ oil\ price}$	
	$Test\ statistic$	$p\ value$
$\boldsymbol{ADF}$	-2.391138	0.3833

4.6 Crude oil prices for the period 1980 to 1995 (First segment)

The first data set from 1980 to 2007 is used to form the first data segment (period 1980-1995). The conclusions from this first segment will be compared with the conclusions of the Garch model with time varying parameters for the exact same period.

4.6.1 Crude oil prices for the period 1980 to 1995 (First segment)

The ACF correlogram of crude oil price for the period from 1980 to 1995 is shown in figure 4.7. The autocorrelation coefficients at various lags are very high. The autocorrelation coefficient starts at a very high level at lag 1(0.984) and declines very slowly. Thus crude oil price time series is non stationary over the period of the first segment.

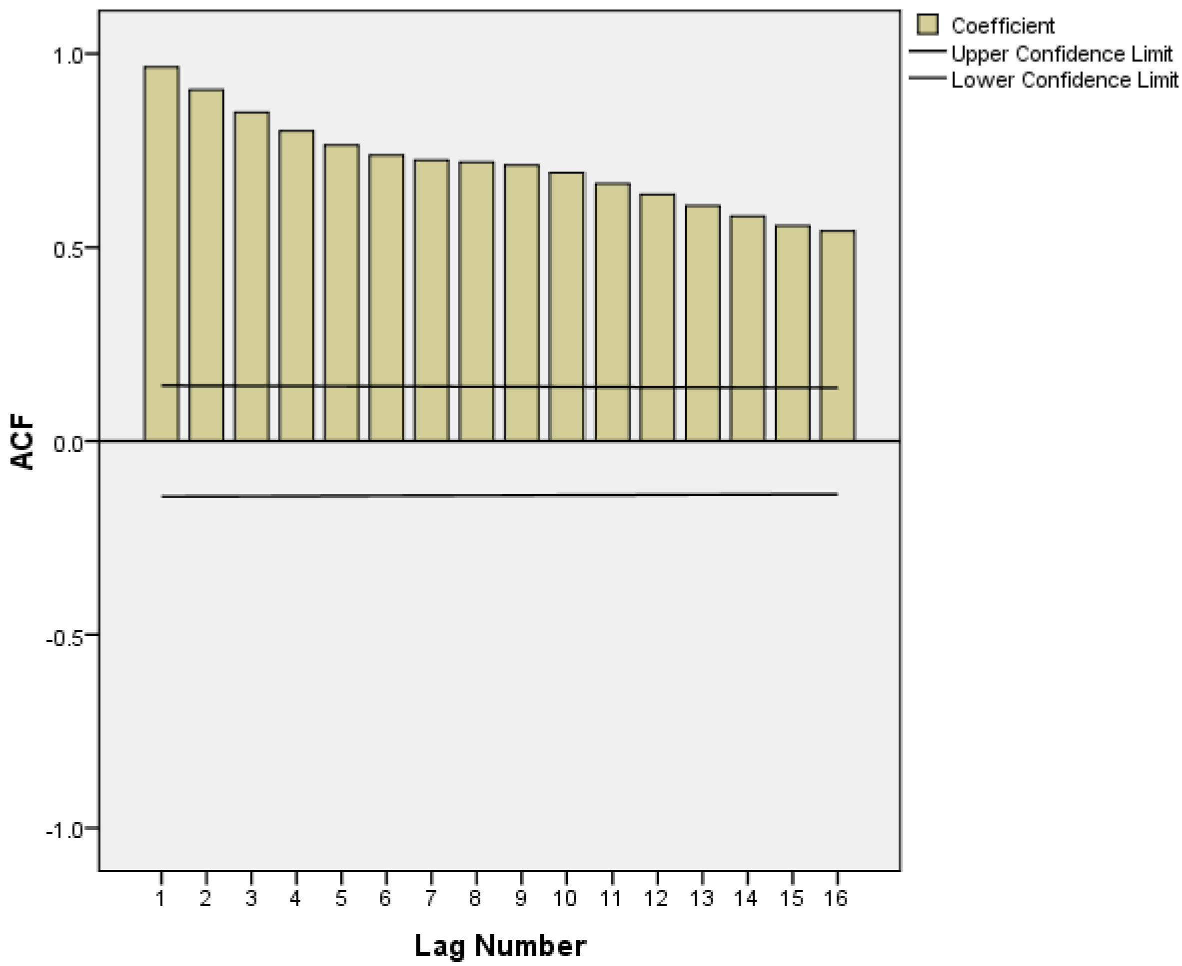

Figure 4.7 Correlogram of crude oil price , 1980 to 1995

The Augmented Dickey-Fuller Test (ADF) test is used to test for random walk or mean reversion for the segment 1980 to 1995.

Considering the first segment; the estimated model for the crude oil price data from 1980 to 1995 is:

$$\Delta P_t = \hat{a}_0 + \beta \hat{P}_{t-1} + a_1 t$$

With the parameter estimates $a_0 = 504222.9$, $(p\ value = 0.0000)$, $\beta = -0.697571$ with $(p\ value = 0.0000)$ and $a_1 = 21.23165$, $(p\ value = 0.0000)$. Thus the equation can be written as:

$$\Delta P_t = 504222.9 - 0.697571 P_{t-1} + 21.23165t$$

Where P_t is the crude oil price. The drift term a_0, slope parameter β and trend parameter a_1 are all significantly different from zero.

$H_o: a_0 = 0 , a_1 = 0, \beta = 0 , series\ is\ non\ stationary\ (has\ a\ unit\ root)$

Table: 4.5 ADF test for crude oil price for the period 1980 to1995

$\boldsymbol{Test}$	$\boldsymbol{Crude\ oil\ price}$	
	$Test\ statistic$	$p\ value$
$\boldsymbol{ADF}$	-10.03604	0.0000

Table 4.5 show the results of ADF test on crude oil price data. Since the test statistic of crude oil price is more negative than the critical values at 5% level, the null hypothesis of non-stationary (unit root) is strongly rejected implying that oil prices is stationary. This result is rather surprising and is not consistent with the results of the whole data set ranging from 1980 to 2007. This result suggests that crude oil price is mean reverting over the period 1980 to 1995 according to Geman (2007:p 233) since the parameter $\hat{\varphi} = \hat{\beta} + 1 = 0.302429$ is significantly different from 1 (unit root).

4.6.2 Log crude oil price for the period 1980 to 1995 (First segment)

In this section we again repeat the same tests for log crude oil price for the same period data. Figure 4.8 shows the ACF correlogram of the log crude oil price for the period 1980 to 1995. All autocorrelation coefficients are high with is a slow decline as from lag 1 indicating non stationarity. Thus the log crude oil price seems to follow a random walk for the period 1980 to 1995.

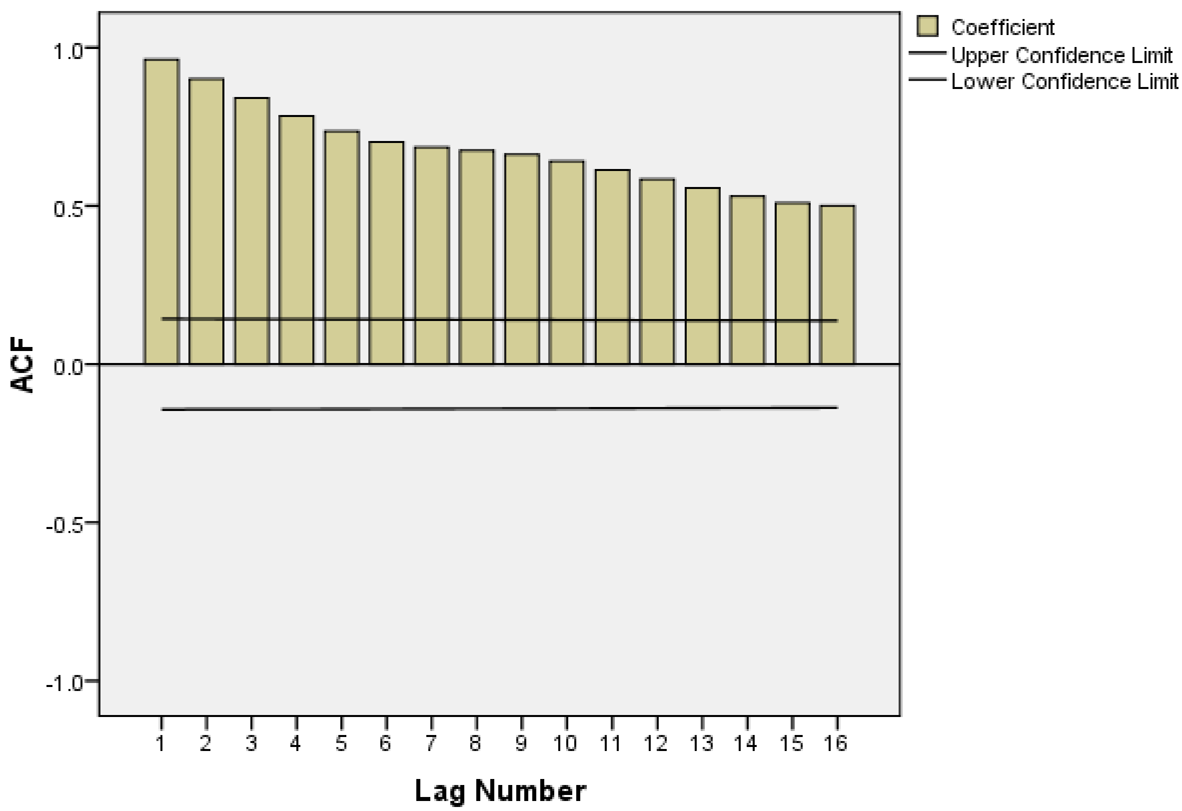

Figure 4.8 Correlogram of log crude oil price, 1980 to 1995

The Augmented Dickey-Fuller Test (ADF) test for log crude oil price for the segment 1980 to 1995 is presented below. The estimated model for the log crude oil price is:

$$r_t = \Delta p_t = \hat{a}_0 + \hat{\beta} p_{t-1} + \hat{a}_1 t \ .$$

where the estimates $a_o = 1.287629$ $(p\ value = 0.0028)$, the drift term is significantly different from zero. The coefficient $\beta = -0.095441(p\ value = 0.0028)$, is significantly different from zero. The trend parameter is insignificantly different from zero. The model can be written as:

$$r_t = \Delta p_t = 1.287629 - 0.095441 p_{t-1}$$

$H_o: a_0 = 0\ , a_1 = 0, \beta = 0\ , series\ is\ non\ stationary\ (has\ a\ unit\ root)$

Table: 4.6 ADF test for log crude oil price for the period 1980 to1995

$\boldsymbol{Test}$	$\boldsymbol{Log\ crude\ oil\ price}$	
	$Test\ statistic$	$p\ value$
$\boldsymbol{ADF}$	-3.031241	0.1265

The log crude oil price $p\ value =\ 0.1265$ is less than 0.05 meaning that the null hypothesis of unit root is not rejected implying that the log crude oil price are thus a random walk. This result is consistent with the ACF results.

The first data series from 1980 to 2007 when analyzed as a whole, the conclusion is that the crude oil price follows a random walk model. However a shorter period called the first segment, we conclude that crude oil price is mean reverting over the period from 1980-1995. However if the data is log transformed over the period 1980-1995, the conclusion is that the log crude oil price follows a random walk.

4.7 Crude oil prices for the period 1995 to 2010 (Second segment)

The second segment of the data is formed from the second data set. Consider Figure 4.9 which presents the correlogram of the crude oil prices from 1995 to 2010. The ACF correlogram is a typical correlogram of a non stationary time series.

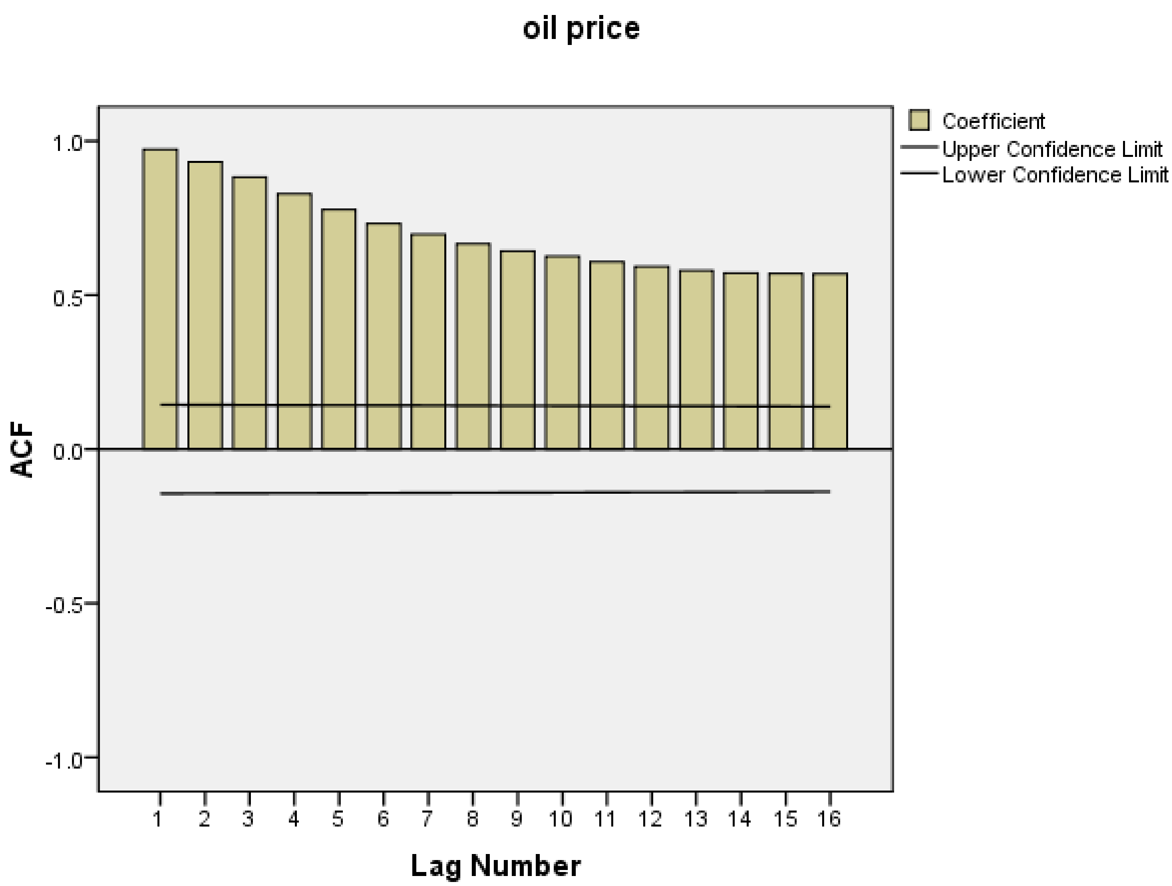

Figure 4.9 Correlogram of crude oil price, 1995 to 2010

The Augmented Dickey-Fuller Test (ADF) test is used to test for the random walk for the model:

The estimated model for crude oil price data from 1995 to 2010:

$$\Delta P_t = \hat{a}_0 + \hat{\beta} P_{t-1} + \hat{a}_1 t$$

where the drift parameter estimates, $a_0 = 0.222112$ ($p\ value = 0.7661$) is insignificantly different from zero, the slope parameter estimate, $\beta = -0.057879$ ($p\ value = 0.0170$) is significantly different from zero and the trend parameter estimate, $a_1 = 0.023809$ is significantly different from zero. The model can be rewritten as:

$$\Delta P_t = -0.029624 P_{t-1} + 0.023809t$$

$H_o: a_0 = 0\,, a_1 = 0, \beta = 0\,, series\ is\ non\ stationary\ (has\ a\ unit\ root)$

Table: 4.7 ADF test for crude oil price for the period 1995 to 2010

$\boldsymbol{Test}$	$\boldsymbol{Crude\ oil\ price}$	
	$Test\ statistic$	$p - value$
$\boldsymbol{ADF}$	-2.407577	0.3744

The value of the ADF test statistic is -2.407577($p\ value = 0.3744$), that is the hypothesis of a random walk is not rejected and the monthly crude oil price follows a simple random walk for the period 1995 to 2010.

4.7.1 Log crude oil price for the period 1995 to 2010

Figure 4.10 shows the ACF of the log crude oil price. The autocorrelation coefficients are declining slowing as lag length increases indicating that log crude oil price is non stationary.

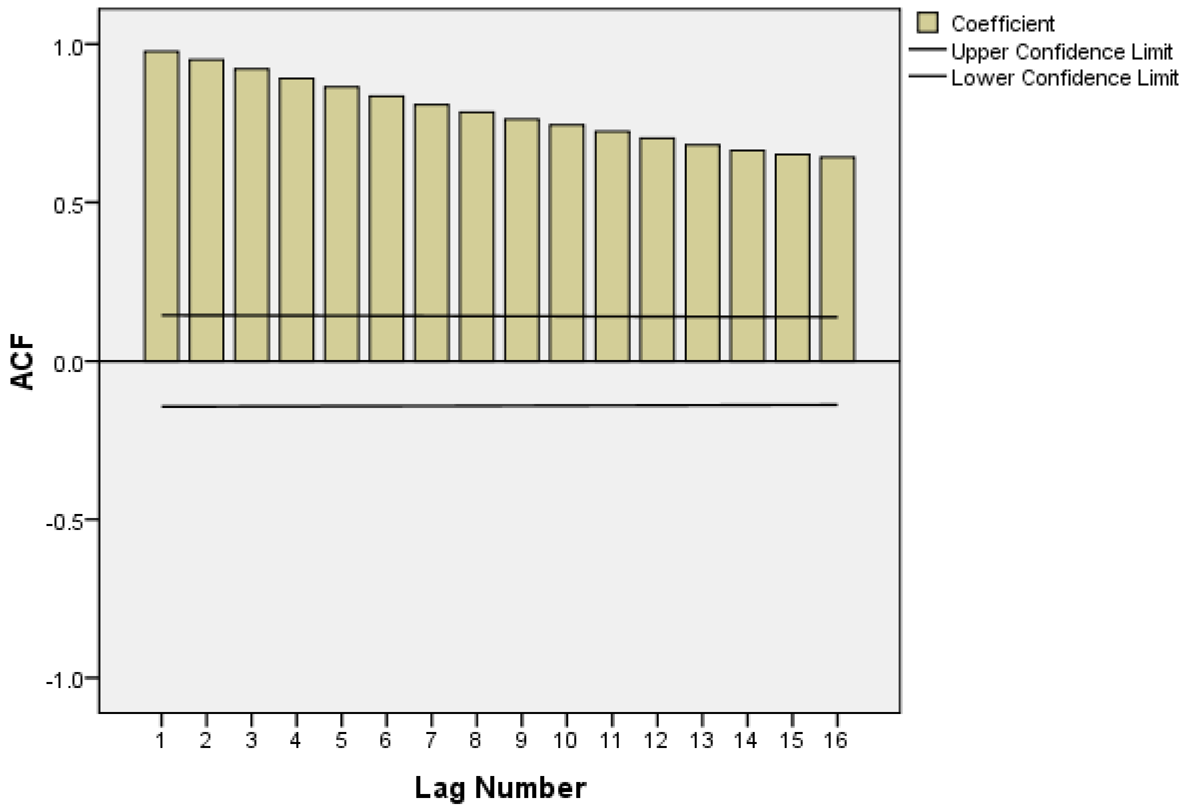

Figure 4.10 Correlogram of log crude oil price, 1995 to 2010

The Augmented Dickey-Fuller Test

The ADF test is used to test for the random walk of the model. The estimated model for the log crude oil price is:

$$r_t = \Delta p_t = \hat{a}_0 + \hat{\beta} p_{t-1} + \hat{a}_1 t$$

where drift parameter estimate, $a_0 = 0.170437$ ($p\ value = 0.0107$) which is significantly different from zero and estimate $\beta = -0.066384$ ($p\ vlue = 0.0000$) which is significantly different from zero. The trend parameter estimate $a_1 = 0.000690$ ($p\ value = 0.0171$) ,which is significantly different from zero. The model can be rewritten as:

$$r_t = \Delta p_t = 0.170437 - 0.066384 p_{t-1} + 0.000690 t$$

$H_o: a_0 = 0, a_1 = 0, \beta = 0$, *series is non stationary (has a unit root)*

Table: 4.8 ADF test for log crude oil price for the period 1995 to 2010

Test	***Log crude oil price***	
	Test statistic	*p value*
ADF	−2.584501	0.2879

The ADF test statistic of -2.584501 ($p\ value = 0.2879$) is smaller than the critical value at 5% significant level, the null hypothesis of unit root is not rejected. Thus the log crude oil price is stationary implying a random walk for log crude oil price. This result is consistent with the ACF test results.

The second data set when considered as a whole (from 1988 to 2010), the conclusion is that the crude oil price and log crude oil price follow a random walk.

These results of the random walk or mean reversion seem to depend on the period under consideration and whether the data is log transformed or not. The behavior of a random walk is more pronounced in crude oil log price (Geman, 2007). In the first segment, we concluded that crude log oil price is a random walk over the period from 1980-1995 after a log transformation of the data but for the untransformed data the conclusion was that crude oil price is mean reverting.

4.8 Garch model with time varying parameters

The results of using the Garch model with time varying parameter are presented in this section. Figures 4.11, 4.12, 4.13, 4.14 and 4.15 present the results of the changes towards / from the random walk. The figures show the paths of the estimated $\beta_{it}{}'s$, $i = 0,1,2$ coefficient (see equation (3.15)) with their respective 95 per cent confidence bands.

For the period 1980 to 2007, the best model using Box Jenkins methodology was an AR (2) model:

$$r_t = \beta_{0t} + \beta_{1t} r_{t-1} + \beta_{2t} r_{t-2} + \mu_t$$

4.8.1 Garch model with time varying parameters (period from 1980 to 2007)

Consider Figure 4.11, which represents the results of the estimated drift parameter $\hat{\beta}_{0t}$ for the period 1980 to 2007. The estimate, $\hat{\beta}_{0t}$, has constant value of – 0.0072 and is just significant and of little practical importance.

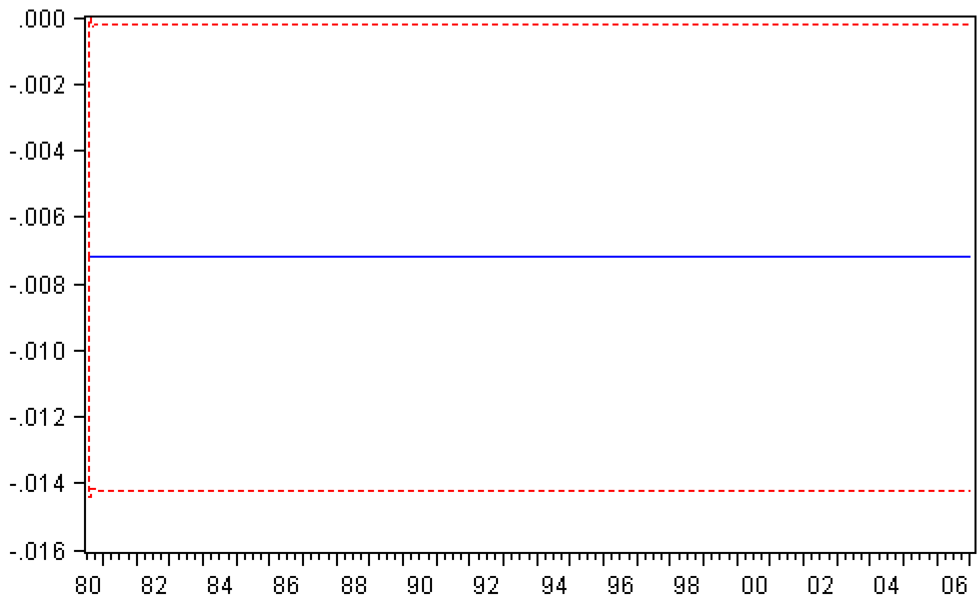

Figure 4.11: $\widehat{\boldsymbol{\beta}}_{0t}$, drift parameter for crude oil 1980 to 2007

Figure 4.12 shows the results of the parameter $\hat{\beta}_{1t}$ for the period 1980 to 2007. The estimate $\hat{\beta}_{1t}$ has an initial value of 0.39 and is significantly different from zero at 0.05 level. The magnitude of the estimated parameter gradually declines and first becomes insignificantlly different from zero in March 1996. The parameter remains insignificant for the rest of the period to end at a level of 0.12 in January 2007.

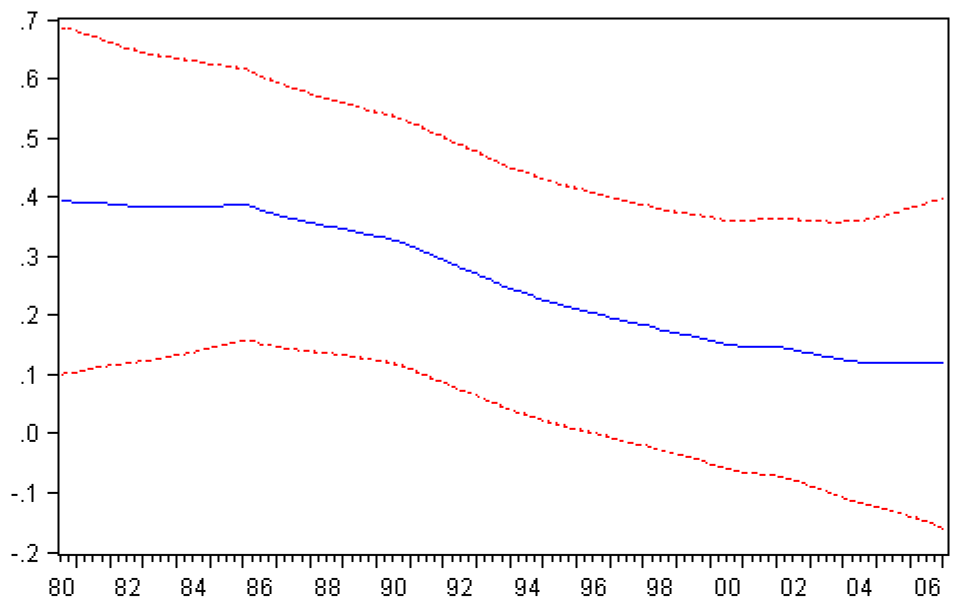

Figure 4.12: $\widehat{\boldsymbol{\beta}}_{1t}$ for crude oil price 1980 to 2007

Figure 4.13 shows the results of the parameter $\hat{\beta}_{2t}$ for the period 1980 to 2007. The estimate $\hat{\beta}_{2t}$ has an initial value of -0.36 and is significantly different from zero at 0.05 level. The magnitude of the estimated parameter gradually increase and first becomes insignificantlly different from zero in March 1996. The parameter remains insignificant for the rest of the period to end at -0.17 in January 2007.

Crude oil price follow the random walk from March 1996. Prior to the year 1996, the finding is that crude oil prices did not follow the random walk i.e. crude oil price were mean reverting.

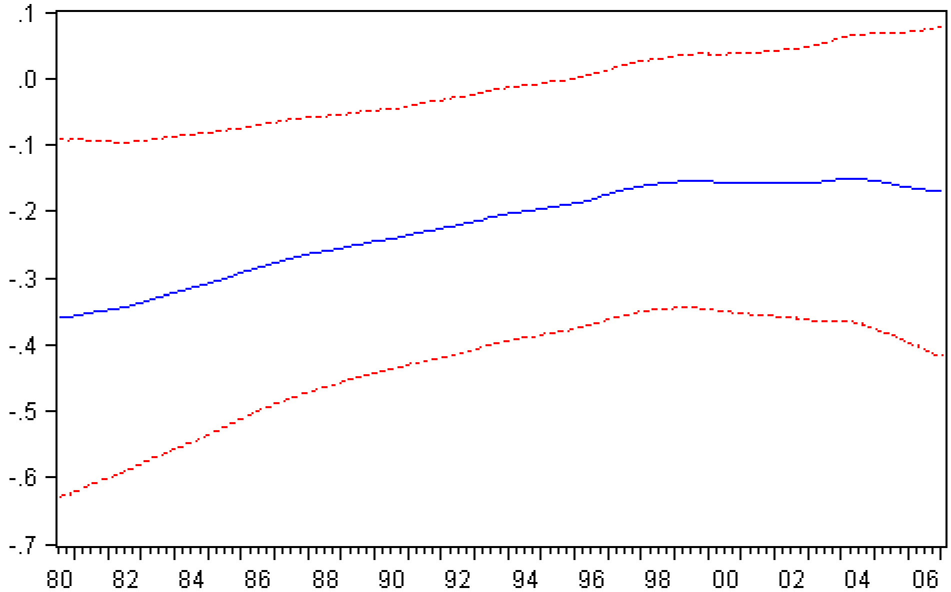

Figure 4.13: $\hat{\boldsymbol{\beta}}_{2t}$ for crude oil price 1980 to 2007

4.8.2 Garch model with time varying parameters (period from 1995 to 2010)

The same AR (2) is imposed on the second data set (period from 1995 to 2010). Figure 4.14 shows the results of the estimated drift parameter $\hat{\beta}_{0t}$ for the period 1995 to 2010. The estimate $\hat{\beta}_{0t}$ has a constant value of 0.101 and is insignificantly different from zero considering its 95 percent confidence limits. The magnitude of the estimated parameter is constant.

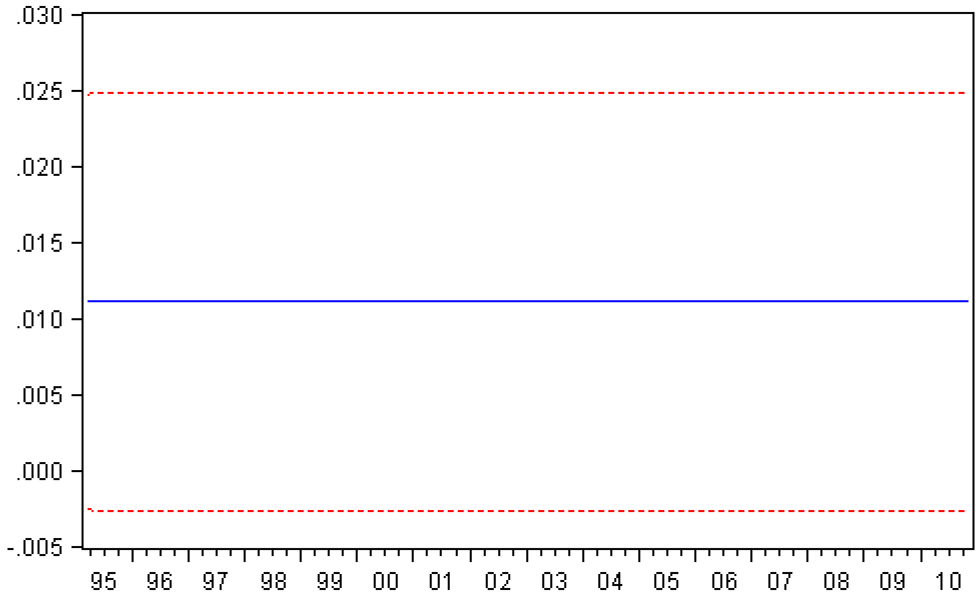

Figure 4.14: $\boldsymbol{\beta_{0t}}$ for crude oil price 1995 to 2010

Figure 4.15 presents the results of the parameter $\hat{\beta}_{1t}$. The estimate of $\hat{\beta}_{1t}$ has an initial value of -0.01 ans is insignificantly different from zero at 0.05 level. The magnitude of the parameter gradually increases but remain insignificant for the period. The same results are found for higher order parameters. Crude oil prices followed the random walk from January 1995 until the end of the study period of November 2010.

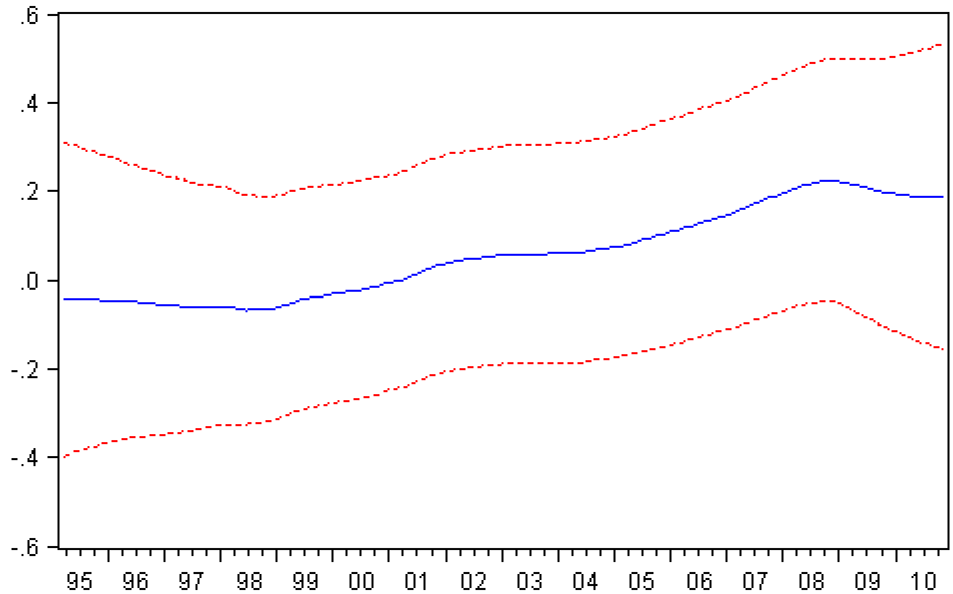

Figure 4.15: The $\widehat{\beta}_{1t}$ for crude oil price 1995 to 2010

A higher order model for the period 1995 to 2010 has no significant parameters, $\beta_{it}'s$.

In summary, the Garch model with time-varying parameters approach using log transformed data shows that crude oil price is mean reverting for the period of January 1980 to February 1996 and follows a random walk for the period March 1996 to November 2010.

CHAPTER 5

Conclusion and Recommendations

5.1 Conclusion

In this study, an attempt was made to determine whether crude oil price is mean reverting or a random walk process. Two approaches namely the Augmented Dickey-Fuller test (ADF) and the Garch model with time-varying properties are used. Before carrying out formal Augmented Dickey-Fuller tests (ADF), the autocorrelation function (ACF) correlogram of crude oil price and log crude oil price were examined to investigate stationarity.

Two data sets, namely for period January 1980 to January 2007 and period January 1988 to November 2010 were used in this study. The first data set was used to form a segment, period ranging from 1980 to 1995. The second data set was used to form a segment from 1995 to 2010.

The first data series from 1980 to 2007 shows evidence of a random walk process yet a shorter period (first segment) shows mean reversion for the period 1980 to 1995 according to the ADF test. The test also shows that crude oil price follow a random walk over the period 1995 to 2010. Thus the results seem to depend on the period under consideration and this is rather puzzling. These results show that the ADF test approach has a limitation of depending on the period under consideration.

The Garch model with time-varying parameters approach shows the presence of mean reversion in log crude oil prices over the period January 1980 to February 1996. It shows a random walk as of March 1996. This approach does not depend on period under consideration and is better than the ADF test. However, the two approaches used in the study, show almost similar result considering the segmented data sets.

The results obtained in this study are similar to the results by Geman (2007) who concluded that, the crude oil price follow a random walk for the period January 1999 to October 2004, using the ADF test. The result of the study also show some similarity with Bessembinder $et\ al$ (1995), who cornfirms the exsistence of mean reversion over the period 1982 to 1999. However, Bessembinder $et\ al$ (1995) differ with the results of this study over the period 2000 to 2005. The authors obtained inconclusive results over the period 2000 to 2005 and in this study; a random walk prevails over that period.

This study used more current monthly data on crude oil prices up to November 2010. The time-varying property approach used in this study produced almost similar results with the ADF test commonly used in investigating mean reversion and random walk. The results are similar only when the data is segmented after observing information from the Garch model. This study concludes the existence of mean reversion for crude oil price over the period 1980 to 1995 and a random walk as of March 1996. The results also confirm a finding by German (2007) that the behavior of a random walk is more pronounced when using oil log price. The ADF test using untransformed data shows that crude oil price is mean reverting for the period of January 1980 to February 1995 yet the log transformed data shows a random walk over the same period.

5.2 Recommendations

The study recommends further study in:

- Using time-varying parameters approach to investigate mean reversion and random walk for assets prices such as exchange rates and prices of precious metals such as gold.
- Using time-varying parameters approach to investigate mean reversion and random walk in futures price of crude oil.

Statisticians and econometricians should use both ADF tests and Time-varying parameter approach to investigate whether crude oil price is mean reverting or a random for period under considering before predicting models for crude oil prices.

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