# PERFORMANCE OF CONFIDENCE INTERVALS ON THE AMONG GROUP VARIANCE IN THE UNBALANCED ONE-FACTOR RANDOM EFFECTS MODEL

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**ABSTRACT**

The objective of this paper is to simultaneously compare the performance of six methods for constructing approximate confidence intervals on the among-group variance component in a simulation study by using their ability to maintain the stated confidence coefficient as criteria. Results suggest that the generalized confidence interval performs well in all designs considered.

## 1. INTRODUCTION

The estimation of variance components serves as an integral part of the evaluation of variation, and is of interest and required in a variety of applications. Estimation of the among-group variance components is often desired for quantifying the variability and effectively understanding these measurements. Much research has been conducted to develop confidence intervals on variance components and articles concerning the topic are spread throughout the literature. However, an exact confidence interval on the among-groups variance component in unbalanced one-factor random models has yet to be reported in statistical literature. A variety of approximate intervals have been reported, discussed and compared in recent years.

In this paper we focus on comparing six confidence intervals on the among-groups variance component in the unbalanced one-factor random model. In section 2 we present the model and introduce notation. Approximate intervals as proposed by Thomas and Hultquist (1978), Khuri (1999), Burdick and Graybill (1992), Burdick and Eickman (1986), Ting et al. (1990), and a generalized confidence interval method proposed by Park and Burdick (2003) are presented in section 3. A simulation study is employed to compare the proposed methods in section 4.

## 2. MODELS AND NOTATION

The model considered is the normal based random effects one-factor design. Although notationally simple, this model has proven useful to practitioners in a variety of fields and results can be generalized to any mixed model with two random error terms. The unbalanced random model is written as

$$Y_{ij} = \mu + A_i + E_{ij} \tag{2.1}$$

Where $i = 1,\ldots,g$, $j = 1, \ldots, n_i$, $\mu$ is an unknown constant, $A_i$ and $E_{ij}$ are mutually independent normal random variables, where the $A_i$'s are distributed $N(0, \sigma_A^2)$, and the $E_{ij}$'s are distributed $N(0, \sigma_E^2)$.

The analysis of variance for the model given in (2.1) is shown in Table 2.1.

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Table 2.1 Analysis of variance for unbalanced one-factor model

| SV | DF | SS | MS | EMS |
|---|---|---|---|---|
| Among groups | $n_A$=g-1 | $SSA = \sum_i n_i(\overline{Y}_{i.} - \overline{Y}_{..})^2$ | $S_A^2 = \dfrac{SSA}{g-1}$ | $\theta_A = \sigma_E^2 + n_0\sigma_A^2$ |
| Within groups | $n_E$=N-g | $SSE = \sum_i \sum_j (\overline{Y}_{ij} - \overline{Y}_{i.})^2$ | $S_E^2 = \dfrac{SSE}{N-g}$ | $\theta_E = \sigma_E^2$ |

where $\overline{Y}_{i.} = \sum_j Y_{ij}/n_i$, $\overline{Y}_{..} = \sum_i \sum_j Y_{ij}/N$, $N = \sum_i n_i$, $n_0 = \dfrac{N - (\sum_i n_i^2/N)}{(g-1)}$

The statistics { $\overline{Y}_{1.}$, $\overline{Y}_{2.}$,..., $\overline{Y}_{g.}$, $S_E^2$ } are sufficient statistics. The mean squares $S_A^2$ and $S_E^2$ are independent and $SSE/\sigma_E^2$ has a chi-square distribution with N-g degrees of freedom. Thus, the confidence interval on $\sigma_E^2$ for the balanced model remains valid. However, design unbalancedness creates a problem concerning the distribution of $SSA/\theta_A$. For the unbalanced model it can be shown that $SSA/\theta_A$ follows a chi-square distribution with g-1 degrees of freedom if and only if $\sigma_A^2 = 0$. Thus, intervals concerning $S_A^2$ for the balanced model are not strictly valid in the unbalanced case.

Formulas in this paper will be presented in a general form such that values of $\alpha_L$ and $\alpha_U$ may be selected according to the confidence interval of interest, i.e. one-sided, two-sided with equal tails or two-sided with unequal tails. For any predetermined value of α, the values of $\alpha_L$ and $\alpha_U$ will be such that equation $\alpha_L + \alpha_U = \alpha$ will hold. For the one-sided upper confidence interval $\alpha_L=\alpha$; $\alpha_U=0$, for the 1 - α one-sided lower confidence interval $\alpha_U = \alpha$; $\alpha_L = 0$, and for equal tails two-sided confidence intervals $\alpha_L = \alpha_U = \alpha/2$. Formulas may also be implemented for unequal tails two-sided confidence intervals where $\alpha_L + \alpha_U = \alpha$, and values for $\alpha_L$ and $\alpha_U$ may be different.

A confidence interval [L, U] that satisfies $P[L \le \theta \le U] = 1 - \alpha$ is called an exact 1 - α confidence interval. Often exact 1 - α confidence intervals do not exist and $P[L \le \theta \le U]$ is only approximately equal to 1 - α. These intervals are referred to as approximate intervals. An approximate interval is conservative if $P[L \le \theta \le U] > 1 - \alpha$ and liberal if $P[L \le \theta \le U] < 1 - \alpha$. As a general rule, conservative intervals are preferred when only approximate intervals are available. However, if it is known that the actual confidence coefficient of a liberal interval is not much below 1 - α, the liberal interval can be recommended. Burdick and Graybill (1992) defined a "good" confidence interval to be one that has a confidence coefficient equal to or close to a specified 1 - α value and thus provides useful information about the parameter of interest.

## 3. CONFIDENCE INTERVALS ON $\sigma_A^2$

Six methods for constructing approximate confidence intervals for the among-group variance component for the unbalanced random one-way model will be discussed and compared.

### 3.1 The Method by Thomas and Hultquist

Thomas and Hultquist (1978) proposed a confidence interval for the among-group variance in the unbalanced case. The unweighted sum of squares associated with $A_i$ in the unbalanced random model (2.1) is $(g-1)S_U^2 = SSU = n_H \sum_i (\overline{Y}_{i.} - \overline{Y}_{.}^{*})^2$, where $\overline{Y}_{.}^{*} = \sum_i \overline{Y}_{i.}/g$, and $n_H = g/\sum 1/n_i$ is the harmonic mean of $n_i$'s. Thomas and Hultquist (1978) showed that $SSU/(n_H\sigma_A^2 + \sigma_E^2)$ has approximately a chi-squared distribution with g-1 degrees of freedom. Using this fact they proposed the following approximate 100(1 - $\alpha_L$)% and 100(1 - $\alpha_U$)% lower and upper confidence bounds for $\sigma_A^2$ :

$$L = \frac{S_U^2 - S_E^2 F_{\alpha_L, g-1, N-g}}{n_H F_{\alpha_L, g-1, \infty}} \text{ and } U = \frac{S_U^2 - S_E^2 F_{1-\alpha_U, g-1, N-g}}{n_H F_{1-\alpha_U, g-1, \infty}} \tag{3.1}$$

We refer to this interval as the TH confidence interval on $\sigma_A^2$. Thomas and Hultquist (1978) found that the approximation with the interval in (3.1) is adequate except in cases where the ratio of the among- and within-group variance $(\lambda_A = \sigma_A^2/\sigma_E^2)$ is small (less than 0.25) and the design is extremely unbalanced. In designs that have a few group sizes of either 1 or 2, TH performs poorly for both small and moderate values of $\rho_A$. This occurs because the exact distribution of $(g-1)S_U^2$ is a weighted sum of (g-1) independent chi-square random variables. As discussed by Burdick and Graybill (1984), these weights are reciprocals of the sample sizes and intermediate values of the sample sizes. Thus, if a few samples of size 1 or 2 are included in the design, the weights on the sum of chi-squared variables will be very unequal, while the TH method assumes equal weights and approximates $(g-1)S_U^2/E(S_U^2)$ with a chi-square distribution. Hence, when the weights are very unequal, the approximation will not perform well.

### 3.2 The Modified Harmonic Mean Method

Khuri (1999) proposed an alternative value to the harmonic mean, $n_H$, used in the TH confidence interval on $\sigma_A^2$. This value, denoted my $n_M$, is given by

$$n_M = \frac{2}{\lambda_{(1)} + \lambda_{(g-1)}},$$

where

$\lambda_{(1)} \geq \lambda_{(2)} \geq \ldots \geq \lambda_{(g-1)}$ are the ordered eigenvalues of the matrix $(I_g - (1/g)J_g)K(I_g - (1/g)J_g)$ and K represents the diagonal matrix, $\text{diag}(1/n_1, 1/n_2, \ldots, 1/n_g)$.

Using $n_M$ instead of $n_H$ in the TH confidence interval, we obtain

$$L = \frac{n_M(S_U^2/n_H) - S_E^2 F_{\alpha_L, g-1, N-g}}{n_M F_{\alpha_L, g-1, \infty}} \text{ and } U = \frac{n_M(S_U^2/n_H) - S_E^2 F_{1-\alpha_U, g-1, N-g}}{n_M F_{1-\alpha_U, g-1, \infty}} \tag{3.2}$$

This interval is referred to as the modified harmonic mean (MHM) confidence interval on $\sigma_A^2$. According to Lee and Khuri (2002) using $n_M$ instead of $n_H$ can provide a slightly better approximation of the distribution of $S_U^2$ as a scaled chi-squared variate, particularly in cases where the design is extremely unbalanced.

### 3.3 The Modified Large Sample Interval

It is known that $SSA = (g-1)S_A^2$ is distributed as a scaled chi-squared variate if the data set is balanced. If, however, the data set is unbalanced, then SSA is distributed as a scaled chi-squared variate if and only if $\sigma_A^2 = 0$. Thus if $\sigma_A^2$ is expected to be close to zero, then it would be appropriate to treat SSA as an approximate scaled chi-squared variate.

Burdick and Graybill (1992) used this idea to construct an approximate confidence interval for $\sigma_A^2$ by modifying the corresponding balanced confidence interval. The resulting approximate 100(1 - $\alpha_L$)% and 100(1 - $\alpha_U$)% lower and upper confidence bounds for $\sigma_A^2$ are then given as

$$L = \frac{S_A^2 - S_E^2 - \sqrt{V_L}}{n_0} \quad \text{and} \quad U = \frac{S_A^2 - S_E^2 + \sqrt{V_U}}{n_0} \tag{3.3}$$

where

$$n_0 = \frac{1}{g-1}\left(N - \frac{1}{N}\sum_{i=1}^{g} n_i^2\right),$$

$$V_L = G_1^2 S_A^4 + H_2^2 S_E^4 + G_{12} S_A^2 S_E^2,$$

$$V_U = H_1^2 S_A^4 + G_2^2 S_E^4 + H_{12} S_A^2 S_E^2,$$

$G_1 = 1 - 1/F_{\alpha_L,g-1,\infty}$ ,  $H_1 = 1/F_{1-\alpha_U,g-1,\infty}$ - 1,

$G_2 = 1 - 1/F_{\alpha_L,N-g,\infty}$ ,  $H_2 = 1/F_{1-\alpha_U,N-g,\infty}$ - 1,

$$G_{12} = \frac{(F_{\alpha_L,g-1,N-g} - 1)^2 - G_1^2 F_{\alpha_L:g-1,N-g}^2 - H_2^2}{F_{\alpha_L:g-1,N-g}} \text{ and } H_{12} = \frac{(1 - F_{1-\alpha_U,g-1,N-g})^2 - H_1^2 F_{1-\alpha_U:g-1,N-g}^2 - G_2^2}{F_{1-\alpha_U:g-1,N-g}}.$$

The lower bound in (3.3) is considered equal to zero if $S_A^2/S_E^2 < F_{\alpha_L,g-1,N-g}$ and the upper bound is considered equal to zero if $S_A^2/S_E^2 < F_{1-\alpha_U,g-1,N-g}$. We refer to the interval in (3.3) as the modified large sample (MLS) confidence interval for $\sigma_A^2$. If $\sigma_A^2$ is far from zero, this procedure might result in very liberal intervals.

### 3.4 The Method by Burdick and Eickman

Burdick, Maqsood and Graybill (1986) suggested an interval for the ratio $(\sigma_A^2/\sigma_E^2)$ which overcomes the problem associated with small ratios in the TH procedure. Using this interval, Burdick and Eickman (1986) developed approximate 100(1 - $\alpha_L$)% and 100(1 - $\alpha_U$)% lower and upper confidence bounds for $\sigma_A^2$ given by

$$L = \frac{S_U^2 L_m}{F_{\alpha_L,g-1,\infty}(1 + n_H L_m)} \text{ and } U = \frac{S_U^2 U_m}{F_{1-\alpha_U,g-1}(1 + n_H U_m)} \qquad (3.4)$$

where

$$L_m = \frac{S_U^2}{n_H}\left[F_{\alpha_L,g-1,N-g} S_E^2\right]^{-1} - \frac{1}{\min(n_1, n_2, \ldots, n_g)} \text{ and } U_m = \frac{S_U^2}{n_H}\left[F_{1-\alpha_U,g-1,N-g} S_E^2\right]^{-1} - \frac{1}{\max(n_1, n_2, \ldots, n_g)}.$$

We refer to this interval as the BE confidence interval for $\sigma_A^2$. Burdick and Eickman used computer simulation to show that (3.4) had a confidence coefficient that was generally at least as great as the stated level. Moreover, although (3.4) was generally known to be more conservative than the TH (3.1) interval, the average interval lengths of the two methods never differed by more than 5% in their simulation study.

### 3.5 The Method by Ting et al.

Another confidence interval proposed for $\sigma_A^2 = E(S_M^2) - \sigma_E^2/n_H$ can be based on $S_M^2$ and $S_E^2$. Although this method, proposed by Ting et al. (1990), requires two independent mean squares that follow scaled chi-square distributions, $S_M^2$ and $S_E^2$ closely mimic these conditions.

The Ting et al. (1990) 100(1 - $\alpha_L$)% and 100(1 - $\alpha_U$)% lower and upper confidence bounds for $\sigma_A^2$ are given by

$$L = S_M^2 - (1/n_H)S_E^2 - \sqrt{V_L} \text{ and } U = S_M^2 - (1/n_H)S_E^2 + \sqrt{V_U} \qquad (3.5)$$

where

$n_H = g/\sum 1/n_i$

$V_L = G_1^2 S_M^4 + (1/n_H{}^2) H_2^2 S_E^4 + (1/n_H) G_{12} S_M^2 S_E^2$,  $V_u = H_1^2 S_M^4 + (1/n_H{}^2) G_2^2 S_E^4 + (1/n_H) H_{12} S_A^2 S_M^2$,

$G_1 = 1 - 1/F_{\alpha_L,g-1,\infty}$ ,  $H_1 = 1/F_{1-\alpha_U,g-1,\infty}$ -1,

$G_2 = 1 - 1/F_{\alpha_L,N-g,\infty}$ ,  $H_2 = 1/F_{1-\alpha_U,N-g,\infty}$ -1,

$$G_{12} = \frac{(F_{\alpha_L:g-1,N-g} - 1)^2 - G_1^2 F_{\alpha_L:g-1,N-g}^2 - H_2^2}{F_{\alpha_L:g-1,N-g}} \text{ and } H_{12} = \frac{(1 - F_{1-\alpha_U:g-1,N-g})^2 - H_1^2 F_{1-\alpha_U:g-1,N-g}^2 - G_2^2}{F_{1-\alpha_U:g-1,N-g}}.$$

If negative lower bounds are obtained, these negative bounds are increased to zero. Given the distributional assumptions of the model, the interval in (3.5) is expected to perform well for large values of $\rho_A$.

Park and Burdick (2003) tested the performance of (3.5) on its ability to maintain the stated confidence coefficient. The method of Ting provided a confidence coefficient less than the stated level when $\rho_A$ was small for very unbalanced designs. Thus, in situation where $\rho_A$ is thought to be small, the Ting method was not recommended for extremely unbalanced datasets. In other situations however, this method performed well.

### 3.6 Generalized Pivotal Quantity for $\sigma_A^2$

Park and Burdick (2003) proposed a generalized pivotal quantity for constructing a generalized confidence interval on $\sigma_A^2$ using results provided by Olsen et al. (1976). Olsen proved that the random variables SSE, $Q_1,\ldots,Q_m$ are mutually independent, where

$SSE=\sum_{i=1}^{g}\sum_{j=1}^{n_i}(\overline{Y}_{ij}-\overline{Y}_{i*})^2$ and

$Q_l = Z'E_lZ \sim (\sigma_E^2 + d_l\sigma_A^2)\chi_{\eta}^2$ for l=1, ....,m, (where $E_l$ is the orthogonal projection operator of the eigenspace of $d_l$).

It will therefore follow that $\sum_{i=1}^{m}\frac{Q_l}{\sigma_E^2 + d_l\sigma_A^2} = U$ has a chi-square distribution with (g-1) degrees of freedom.

To construct a generalized confidence interval for $\sigma_A^2$ define T as the solution for $\sigma_A^2$ in the non-linear equation

$$\sum_{i=1}^{m}\frac{q_l}{sse/R_E + d_l\sigma_A^2} = U \tag{3.6.1}$$

where

$q_l$ and sse are observed values of $Q_l$ and SSE, respectively, and $R_E = SSE/\theta_E$ and $U = \sum_l Q_l/(\sigma_E^2 + d_l\sigma_A^2)$ are jointly independent observable chi-square variables with N-g and g-1 degrees of freedom respectively.

Note that the distribution of T is completely determined by the joint distribution of $R_E$ and U and is free of parameters contained in the model. We simulate 10 000 (or more) values of T and sort them from least to greatest. The simulation is performed by simulating $R_E$ and U. If $U > (R_E/SSE)\sum_l q_l$, then let T=0, since $\sigma_A^2 \geq 0$. If $U \leq (R_E/SSE)\sum_l q_l$, then the bisection method can be used to solve the non-linear equation in (3.6.1). The approximate 100(1 - $\alpha_L$)% and 100(1 - $\alpha_U$)% lower and upper confidence bounds for the parameters are

$$L = T_{\alpha_L} \text{ and } U = T_{1-\alpha_U} \tag{3.6}$$

where

$T_{\alpha_L}$ is the value of T in position 10 000 x $\alpha_L$, and $T_{1-\alpha_U}$ is the value of T in position 10 000 x (1 - $\alpha_U$).

Park and Burdick (2003) compared the performance of (3.6) and (3.5) on their ability to maintain the stated confidence coefficient and the average length of the two-sided confidence intervals. The generalized method maintained the stated confidence coefficient across all values of $\rho_A$. In contrast the method of Ting (3.5) provided a confidence coefficient less than the stated level when $\rho_A$ was small. The average interval lengths for both methods however, were very similar. A disadvantage of the generalized confidence interval proposed by Park and Burdick (2003) may be that it requires the numerical solutions of non-linear equations to be computed, which is relatively difficult to carry out.

## 4. SIMULATION STUDY

In order to compare the methods described in section 3 for constructing a confidence interval for $\sigma_A^2$ in the unbalanced one-way random effects model, a simulation study was performed. The criteria for analyzing the performance of the methods are the ability to maintain the stated confidence coefficient, and the average length of the two-sided confidence intervals. Although shorter average interval lengths are preferable, it is necessary that the methods first maintain the stated confidence coefficient. In order to test the methods, three unbalanced patterns were selected for investigation and are shown in Table 4.1.

Table 4.1 Unbalanced designs used for the simulation study

| Pattern | g | **N** | $n_i$ | Degree of Imbalance ( $\phi$ ) |
|---|---|---|---|---|
| 1 | 3 | 30 | 3, 7, 20 | 0.6550 |
| 2 | 6 | 35 | 5, 2, 7, 5, 7, 9 | 0.8763 |
| 3 | 10 | 60 | 1,1, 4, 5, 6, 6, 8, 8, 9, 12 | 0.7692 |

Note that patterns were selected to be similar to patterns used by Burdick and Eickman (1986) and Park and Burdick (2003), with slight adjustments to the $n_i$'s in order to represent a wider spectrum of degree of imbalance.

These unbalanced patterns were used to simulate Y values using the definitions of an unbalanced one-factor random model as given in equation (2.1). The same procedure as proposed by Park and Burdick (2003) was followed. Without the loss of generality, $\sigma_A^2$ was set to $1-\sigma_E^2$ so that $\rho_A = \sigma_A^2$ and $1-\rho_A = \sigma_E^2$. Random variables $A_i$ and $E_{ij}$ were independently generated from normal populations with zero means and variances $\rho_A$ and $1-\rho_A$ respectively. Selected values for $\rho_A$ were 0.001, 0.1, 0.2,…, 0.9, 0.999. For each value of $\rho_A$, 1000 data sets were simulated in each pattern. The generalized confidence interval is based on 10 000 simulated values for each data set. The stated confidence coefficient for all intervals is 90%. Matlab was used to perform all simulations.

Two-sided intervals were computed for each proposed method. Confidence coefficients were determined by counting the number of intervals that contained $\sigma_A^2 = \rho_A$. Using the normal approximation to the binomial, if the true confidence coefficient is 90%, there is a less than 2.5% chance that an estimated confidence coefficient based on 1000 replications will be less than 88.1%. The average lengths of the two-sided intervals were also calculated.

## 5. RESULTS

The results presented in Figures 5.1-5.6 are the simulated confidence coefficients and average interval lengths obtained using the six proposed methods for the three patterns mentioned in Table 4.1.

It is apparent from Figures 5.1-5.3 that the Burdick-Eickman (BE) procedure provides too conservative intervals (that is, the coverage probabilities are larger than the nominal 90% value) when $\rho_A$ is small. The estimated confidence coefficient of this interval declines if $\rho_A$ becomes larger and lies near the nominal value if $\rho_A$ is large. For large values of $\rho_A$, coverage is as good as some of the other procedures. This result is consistent with results found by Burdick and Graybill (1992), Hartung and Knapp (2000) and Lee and Khuri (2002).

The modified large sample (MLS) procedure provides liberal confidence intervals (that is, the coverage probabilities are smaller than the nominal 0.90 value) when $\rho_A$ is far away from zero. This result is most evident for pattern 3 (Figure 5.3). The estimated confidence coefficient of this interval increases if $\rho_A$ becomes smaller. For patterns 1 and 2 (Figures 5.1 & 5.2) true coverage probabilities do not differ severely from the nominal value. This result is also consistent with results found by Burdick and Graybill (1992), and Lee and Khuri (2002). The reason for this is because the MLS interval is based on the assumption that $(g-1)S_A^2/\theta_A$ has a chi-squared distribution with (g-1) degrees of freedom. In the unbalanced design however, this is true if and only if $\sigma_A^2 = 0$. For this reason the MLS interval cannot generally be recommended, unless it is known that $\sigma_A^2$ is close to zero.

The TH, MHM and Tin procedures behave similarly for pattern 1 and 2 (Figures 5.1 & 5.2), by maintaining their coverage probabilities close to the nominal value and never dropping below the 88.1% level. For pattern 3 (Figure 5.3), the TH and Tin procedures still behave similarly but they produce somewhat liberal intervals for small values of $\rho_A$. The MHM procedure, on the other hand, provides extremely liberal confidence intervals for small $\rho_A$, particularly for this pattern. For all three procedures, the estimated confidence coefficient increased as $\rho_A$ became larger. This is due to the fact that $(g-1)S_U^2/\theta_U$ has an exact chi-square distribution only when $\rho_A = 1$. Thus, in situations where $\rho_A$ is thought to be small ( $\rho_A \le 0.4$, say)

TH, MHM and Tin procedures are not recommended for extremely unbalanced datasets. These results are mostly consistent with results found by Lee and Khuri (2002) and Park and Burdick (2003).

Using the above-mentioned normal approximation criterion, it is clear that the generalized confidence interval (GEN) method maintains the stated confidence coefficient across all values of $\rho_A$ for all three patterns. This result is consistent with results found Park and Burdick (2003).

It is apparent from Figures 5.4-5.6 that the MLS procedure provides the shortest interval length. As mentioned by Burdick and Graybill (1992), this is typically the case, but this interval can have a confidence coefficient much less than the stated level. Minor differences in interval lengths of the other procedures may occur because some negative bounds have been increased to zero, but in general the average lengths do not vary much between methods. For pattern 3 (Figure 5.6) there is a greater variation in interval lengths than for the other two patterns.

## 6. CONCLUSIONS

The simulation study confirm results by Burdick and Graybill (1992), and Lee and Khuri (2002) that the Burdick-Eickman (BE) method provides conservative intervals when $\rho_A$ is small, while the TH, MHM and Tin procedures liberal intervals for small values of $\rho_A$ and the modified large sample (MLS) procedure provides liberal confidence intervals when $\rho_A$ is far away from zero. The generalized confidence interval (GEN) is the only method that maintains the stated confidence coefficient across all values of $\rho_A$ for all three patterns and is therefore recommended.

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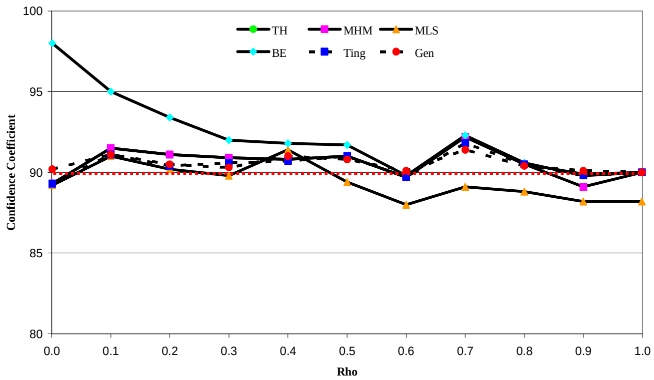


Fig.5.1 Simulated confidence coefficients for 90% intervals on $\sigma_A^2$ for pattern 1.

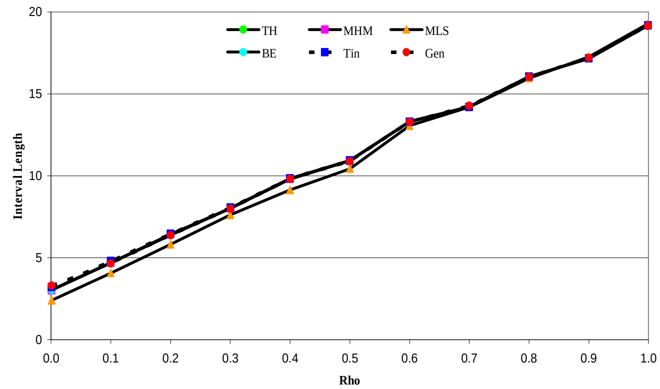


Fig.5.4 Simulated average interval lengths for 90% intervals on $\sigma_A^2$ for pattern 1.

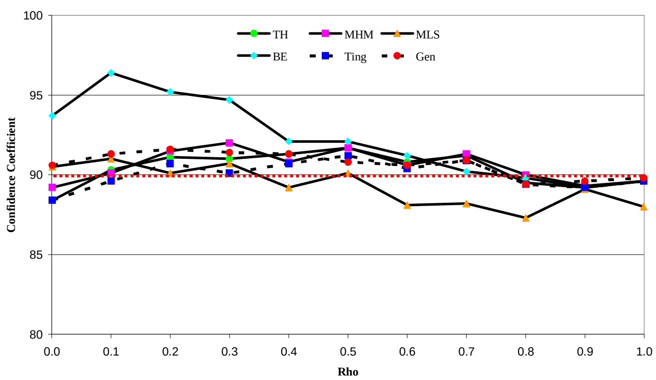


Fig.5.2 Simulated confidence coefficients for 90% intervals on $\sigma_A^2$ for pattern 2.

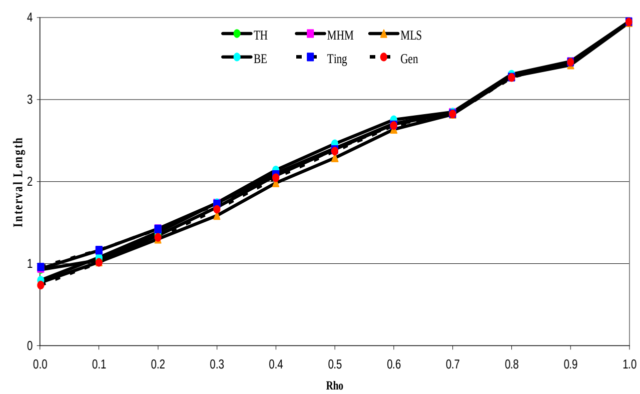


Fig.5.5 Simulated average interval lengths for 90% intervals on $\sigma_A^2$ for pattern 2.

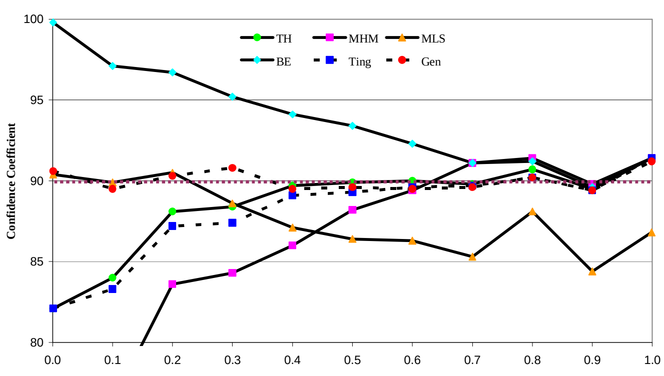


Fig.5.3 Simulated confidence coefficients for 90% intervals on $\sigma_A^2$ for pattern 3.

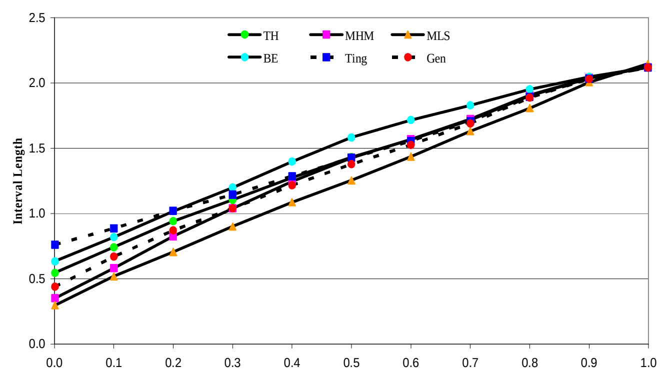


Fig.5.6 Simulated average interval lengths for 90% intervals on $\sigma_A^2$ for pattern 3.